Practice Round Mu School Bowl
Mu Alpha Theta National Convention 2013

P1. As a common fraction, simplify: ln(1024) / ln(64)

P2. Evaluate: $\int_{13}^{37} 2x \ dx$

P3. Find the smallest positive prime factor of 4004.

P4. Evaluate $\frac{dy}{dx}$ for $y = \cos^{2013} x \sec^{2013} x$.

P5. Let A, B, C, and D be the answers to questions P1, P2, P3, and P4, respectively.
Evaluate: $BAD + C$

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Round #1 Mu School Bowl
Mu Alpha Theta National Convention 2013

1. Find the largest integer x such that $|x-2| < |x-6|$.

2. Find the period of the graph of $y = \left|\sin\left(\frac{\pi x}{6}\right)\right|$.

3. Evaluate: $\lim_{x\to 0} \frac{\sin(2013x)}{2013x}$

4. Evaluate: $\lim_{x\to 0} \left(\frac{1}{\sin^2 x} - \frac{1}{x^2}\right)$

5. Let A, B, C, and D be the answers to problems 1, 2, 3, and 4, respectively.
Evaluate: $ABCD^{-1}$

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Evaluate: $ABCD^{-1}$

Round #2 Mu School Bowl
Mu Alpha Theta National Convention 2013

6. How many digits are there in the binary representation of 5566?

7. Let $ABCD$ be a square, with E and F the midpoints of AB and BC, respectively. If $m\angle EDF = \theta$, find $\sin\theta$.

8. Evaluate: $\lim_{x\to\infty} \frac{100}{x^2+1}$

9. Evaluate: $\lim_{x\to\infty} \frac{12x-5}{4+3x}$

10. Let A, B, C, and D be the answers to problems 6, 7, 8, and 9, respectively.
Evaluate: $A^2 - BC + D^2$

Round #2 Mu School Bowl
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Round #3 Mu School Bowl
Mu Alpha Theta National Convention 2013

11. Let $L(x)$ be a linear function of positive slope and $I(x)$ be the inverse of $L(x)$. Given that $L(x) = 4I(x) + 3$ for all real x, find the value of $L(10)$.

12. Given that $\cos(2x) = \frac{3}{7}$, the value of $\cos^2 x = \frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

13. The line $y = mx + b$, for constants m and b, is tangent to the graph of $y = x^2 - 11x + 53$ at $(7, 25)$. Evaluate $m^2 + b^2$.

14. Let f be a function and $f''(x) = 0$ for all x. If $f(0) = 20$ and $f(1) = 13$, find $f(17)$.

15. Let A, B, C, and D be the answers to problems 11, 12, 13, and 14, respectively.
Evaluate: $-A + B - C + D$

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Evaluate: $-A + B - C + D$

Round #4 Mu School Bowl
Mu Alpha Theta National Convention 2013

16. Solve for x: $\frac{x+3}{x+5} = \frac{x+1}{x+2}$

17. Evaluate: $(\sin 20°)(\tan 10° + \cot 10°)$

18. A plane 3 meters away from the center of a sphere with radius 6 meters cuts the sphere into two regions. Find the ratio of the volume of the region containing the center of the sphere to the volume of the sphere. Express your answer as a common fraction.

19. Let f and g be differentiable functions such that $f(3) = 2$, $f'(3) = 4$, $g(5) = 3$, and $g'(5) = 7$. Find the value of the derivative of $f(g(x))$ with respect to x, evaluated at $x = 5$.

20. Let A, B, C, and D be the answers to problems 16, 17, 18, and 19, respectively.
Evaluate: $\frac{(A+B)^3}{C} + \frac{D}{2}$

Round #4 Mu School Bowl
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Evaluate: $\frac{(A+B)^3}{C} + \frac{D}{2}$

Round #5 Mu School Bowl
Mu Alpha Theta National Convention 2013

21. Find the sum of the third hexagonal number and the fourth octagonal number.

22. If (x, y) is an ordered pair of real numbers such that $x^2 + y^2 = 1$, find the maximum value of $2(x + y)^3$.

23. Evaluate: $\int_1^{\pi/2} ((x-1)e^x \cos x - (x-1)e^x \sin x + e^x \cos x)\ dx$

24. Evaluate:

$$\int_1^9 \frac{2x\sqrt{x} - \dfrac{x^2+3}{2\sqrt{x}}}{x}\ dx$$

25. Let A, B, C, and D be the answers to problems 21, 22, 23, and 24, respectively.
Evaluate: $A^2 - B^2 + C^2 - D$

Round #5 Mu School Bowl
Mu Alpha Theta National Convention 2013

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25. Let A, B, C, and D be the answers to problems 21, 22, 23, and 24, respectively.
Evaluate: $A^2 - B^2 + C^2 - D$

Round #6 Mu School Bowl
Mu Alpha Theta National Convention 2013

26. Let vectors $\mathbf{a} = [-1, 0, 1]$, $\mathbf{b} = [3, 4, 3]$, and $\mathbf{c} = [2, -3, 2]$. If $[-6, -17, 6] = c_1\mathbf{a} + c_2\mathbf{b} + c_3\mathbf{c}$, find the value of $c_1c_2c_3$.

27. Find the sum of all $x \in [0, 2)$ such that $\cos(3\pi x) = \sin(2\pi x)$.

28. Let $f(x)$ denote the greatest integer less than or equal to x. Evaluate: $\sum_{n=0}^{9}\left(\int_{n}^{n+2} f(x)\ dx\right)$

29. Region R is defined to be the region in the xy-plane bounded by the x-axis, the line $x = 1$, and the graph of $y = kx^2$, for some positive k. Keith wants Richard to calculate the volume when R is revolved about the x-axis. Richard accidentally revolves R around the y-axis instead. Surprisingly, he obtains the same volume. Find the value of $8k$.

30. Let A, B, C, and D be the answers to problems 26, 27, 28, and 29, respectively.
Evaluate: $\sqrt{A^2} + \sqrt{B + D + 10} + \sqrt{C}$

Round #6 Mu School Bowl
Mu Alpha Theta National Convention 2013

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30. Let A, B, C, and D be the answers to problems 26, 27, 28, and 29, respectively.
Evaluate: $\sqrt{A^2} + \sqrt{B + D + 10} + \sqrt{C}$

Round #7 Mu School Bowl
Mu Alpha Theta National Convention 2013

31. In the Cartesian plane, let O be the origin, $Q = (5, 0)$ and P a point on the circle with equation $x^2 + y^2 = 36$. As P goes along the entire circumference of the circle, the centroid of triangle OPQ traces out a curve C. Find the area enclosed by C.

32. The cosine of the smallest angle in a triangle with side lengths 6, 7, 8 is m/n where m and n are relatively prime positive integers. Find $m + n$.

33. Evaluate: $\int_0^\infty \frac{dx}{1+x^2} = \lim_{b\to+\infty} \int_0^b \frac{dx}{1+x^2}$

34. Evaluate: $\int_0^1 \sqrt{\frac{1-x}{x}}\, dx = \lim_{b\to0^+} \int_b^1 \sqrt{\frac{1-x}{x}}\, dx$

35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $(A - B)(C - D)$

Round #7 Mu School Bowl
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33. Evaluate: $\int_0^\infty \frac{dx}{1+x^2} = \lim_{b\to+\infty} \int_0^b \frac{dx}{1+x^2}$

34. Evaluate: $\int_0^1 \sqrt{\frac{1-x}{x}}\, dx = \lim_{b\to0^+} \int_b^1 \sqrt{\frac{1-x}{x}}\, dx$

35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $(A - B)(C - D)$

Round #8 Mu School Bowl
Mu Alpha Theta National Convention 2013

36. Given that $M = \begin{bmatrix} -29 & -20 \\ 42 & 29 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, find the sum of the entries of M^{10}.

37. Find the number of times the line $y = \frac{x}{100\pi} - 1$ intersects the graph of $y = \sin x$.

38. Let $f(x) = \int_0^{\pi} t^x \sin t \, dt$, where x is a real number. A number c is selected at random from the set $\{2,\ e,\ \sqrt{5},\ \frac{2\pi}{3},\ 2.5,\ 3, \pi\}$. Find the probability the equation $f(x) = c$ has at least one solution.

39. Let $A = (0, -10)$, $B = (2, 0)$, and $C = (c, d)$ be a point on the graph of $y = x^2$. Find the value of c that minimizes the area of triangle ABC.

40. Let A, B, C, and D be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $2AD + B + C$

Round #8 Mu School Bowl
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Evaluate: $2AD + B + C$

Round #9 Mu School Bowl
Mu Alpha Theta National Convention 2013

41. In convex quadrilateral $ABCD$, $|AB| = \sqrt{6}$, $|BC| = 5 - \sqrt{3}$, $|CD| = 6$, $m\angle ABC = 135°$, and $m\angle BCD = 120°$. Find the length of AD in simplest radical form.

42. Let $P(x)$ be the second-degree minimal polynomial with integer coefficients (and positive leading coefficient) such that $P(\cos(\pi/5)) = 0$. Evaluate: $P(-1)$

43. For $f(x) = \frac{2}{729}(4\cos^3 x - 3\cos x)$, let n equal the smallest positive value such that the nth derivative of f evaluated at $x = 0$ is a positive integer. Find n^2.

44. If $P(x)$ is a polynomial function with real coefficients such that $\frac{d}{dx}\left(\frac{P(x)}{x^4+1}\right) = \frac{3x^4-1}{(x^4+1)^2}$ for all x, find $P(15)$.

45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $A^2 + B^2 + (C - D + 1)^2$

Round #9 Mu School Bowl
Mu Alpha Theta National Convention 2013

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45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $A^2 + B^2 + (C - D + 1)^2$

Round #10 Mu School Bowl
Mu Alpha Theta National Convention 2013

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46. If $f(x) = \log_2\left(1 + \sqrt{8x + 1}\right) - 2$, evaluate: $f(6) + f(28) + f(496) + f(8128) + f(33550336)$

47. The side lengths of an isosceles triangle with no right angles are $\csc x$, $\sec x$, and $\cot x$. Find the largest possible value of $\csc x$.

48. If $f(x) = e^2$, find $f'(2013)$.

49. If $f(x) = 3\pi^2$, evaluate $\int_0^3 f(x)\ dx$.

50. Let A, B, C, and D be the answers to problems 46, 47, 48, and 49, respectively.
Evaluate: $A + BC + \cos\sqrt{D}$

Round #10 Mu School Bowl
Mu Alpha Theta National Convention 2013

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Evaluate: $A + BC + \cos\sqrt{D}$