$$A = \int_{3}^{1} (3x^2 - 2x + 2) dx$$

 $B = \text{ the value of } f'(2) \text{ if } f(x) = x^3 - x^2 + x - 2$

$$C = \lim_{x \to -\infty} \frac{37429x^2 - 66x^3 - 12473}{6x^3 + 47923x - 97624}$$

D =the minimum value of $f(x) = x^2 - 2x + 6$

Compute the value of the determinant $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$.

National MAO 2008 Mu School Bowl Question #2

Which of the following statements concerning the graph of $f(x) = ax^3 + bx^2 + cx + d$ are true? Assume a, b, c, and d are all rational and nonzero. (Write the letters of the correct statements on your answer sheet.)

- (A) f(x) has a point of inflection at $x = -\frac{b}{3a}$.
- (B) The distance along the *x*-axis between the two extrema of f(x) is $\frac{\sqrt{b^2 3ac}}{3a}$.
- (C) The graph of f(x) is tangent to the x-axis at x = -1 if and only if 4a 3b + 2c d = 0.
- (D) The leftmost of the two extrema is a local minimum if and only if a > 0.

Let R denote the region of the plane bounded by the graphs of the lines y = 2x + 3, y = 2, x = 0, and x = 1.

A = the volume of the solid formed by revolving R about the line y = -2

B = the volume of the solid formed by revolving R about the line x = -2

Compute A - B.

National MAO 2008 Mu School Bowl Question #4

Let αR^3 = the volume of the largest cylinder that can be inscribed in a sphere of radius R. Find α .

If $f(x) = (3x^4 + 7)^3 (2 - 5x)^6 \cos^5 (x - \frac{\pi}{6})$, then what is the value of f'(0)?

National MAO 2008 Mu School Bowl Question #6

Let βR^3 = the volume of the largest cylinder that can be inscribed in the largest cone that can be inscribed in a sphere of radius R. Find β .

Compute
$$\int_{-5}^{2} |x^3 - 2x^2 - 9x + 18| dx$$
.

Let $f(x) = \sqrt{36 - x^2}$.

$$M = \int_{-6}^{6} f(x) dx$$

A =the area of the region bounded by the y-axis, the graph f(x), and the line $y = x \tan \theta$ for $0 < \theta < \frac{\pi}{2}$

R =the arc length of the curve f(x) from $(-3\sqrt{3},3)$ to $(3\sqrt{2},3\sqrt{2})$.

K = the slope of the curve f(x) at the point where f(x) intersects the line y = bx

Compute the product MARK.

Mrs. Singleton and Mr. Bantz are chatting about their favorite functions. Mrs. Singleton tells Mr. Bantz that her favorite function is a cubic polynomial function with rational coefficients. "Go on," he says, intrigued. Mrs. Singleton continues, "well, it has a point of inflection at (2,-6), but my favorite part is the local maximum at (-1,48)." At this point, Mr. Bantz doesn't need any more clues. "Oh, *that* one," he says, "My wife loves that one." Find the sum of the coefficients of Mrs. Singleton's favorite function.

National MAO 2008 Mu School Bowl Question #10

Find the sum of the positive numbers A and B which satisfy the following equalities.

$$\int_{0}^{A} w e^{w^{2}} dw = 9$$

$$\int_{1}^{B} y \ln y dy = \frac{1}{4} (e^{2} + 1)$$

Let $f(x) = x^2 - 2x - 8$.

A = the value of x guaranteed by Rolle's Theorem on [-2,4]

B = the value of x guaranteed by the Mean Value Theorem for Derivatives on [6,12]

C =the average value of f(x) on [-6,9]

D = the minimum value of f(x)

Compute A + B + C + D.

National MAO 2008 Mu School Bowl Question #12

Each of the following statements concerning the function $f(x) = 3x^4 - 10x^3 - 27x^2 + 108x - 74$ has a value, indicated to the left of each statement. Find the sum of the values of the correct statements.

- (-4) f(x) has an x-intercept at x = 1.
- (1) f(x) has a point of inflection at (2,2).
- (0) f(x) has a local minimum at (-2,-270).
- (5) f(x) has a y-intercept at y = -74.
- (-2) f(5) = 376

Compute
$$\int_{\frac{3}{2}}^{\frac{9}{4}} \frac{dx}{\sqrt{3x - x^2}}.$$

$$A = \lim_{h \to 0} \frac{f(1) - f(1-h)}{h}$$
 for $f(x) = \ln x$

$$B = \lim_{h \to 0} \frac{g\left(\frac{\pi}{6} + h\right) - g\left(\frac{\pi}{6}\right)}{h} \text{ for } g(x) = \cot x$$

$$C = \lim_{y \to 3} \frac{h(y) - h(3)}{y - 3} \text{ for } h(x) = x^3 + 2x^2 + 3x + 4$$

Find A + B + C.

Compute
$$\int_{\frac{\pi}{6}}^{\frac{5\pi}{4}} e^{4x} \cos 5x dx.$$