Mu Alpha Theta National Convention: Denver, 2001

Denver Bowl Solutions – Mu Division

1. x^2 & y^2 have different signs: hyperbola

2. $\frac{2}{\frac{1}{3}+\frac{1}{5}} = \frac{15}{4}$

3. $\frac{\ln 81}{\ln 8} \cdot \frac{\ln 125}{\ln 9} \cdot \frac{\ln 64}{\ln 5}$

$\frac{4\ln 3}{3\ln 2} \cdot \frac{3\ln 5}{2\ln 3} \cdot \frac{6\ln 2}{\ln 5} = 12$

4. $s'(3) = 9t^2 + \frac{18}{t^2}\Big|_{t=3} = 81+2 = 83$

5. $\int_0^2 (x-1)^3 - 1$

$\frac{1}{4}(x-1)^4 - x \Big|_0^2 = \frac{1}{4} - 2 - (\frac{1}{4} - 0)$

$= -2$

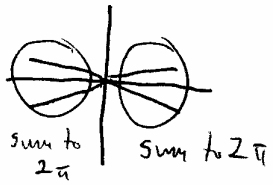
6. 

$A = \frac{1}{2}Pr = \frac{1}{2} \cdot 128 \cdot 2\sqrt{2}$

$= 128\sqrt{2}$

7. $\cos\theta = \pm\sqrt{\frac{3}{5}}$

4π



8. $\log 4 = a \Rightarrow \log 2 = \frac{a}{2}$

$\log 20 = \log(2\cdot 10) = \log 2 + \log 10$

$= \frac{a}{2} + 1$

9. $\begin{vmatrix} 1-\lambda & 4 \\ -2 & 8-\lambda \end{vmatrix} = \lambda^2 - 9\lambda + 8 - 8$

$= \lambda^2 - 9\lambda + 16$

product of roots $= \frac{c}{a} = 16$

10. $(2h-1)(3h-2) = (h+4)(h+3)$

$6h^2 - 7h + 2 = h^2 + 7h + 12$

$5h^2 - 14h - 10 = 0$

sum of roots $= \frac{-b}{a} = \frac{14}{5}$

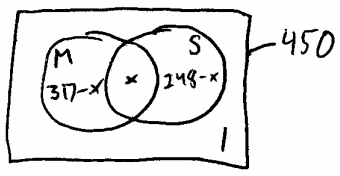
11. #red = #blue = x

$\frac{x \cdot x}{\binom{4}{2}} = \frac{4}{7} = \frac{2x^2}{2\times 4\times 7} = \frac{8x^2}{2x^2 - x}$

$8x^2 - 4x = 7x^2$

$x^2 - 4x = 0$

$x(x-4) = 0 \Rightarrow x = 4 \Rightarrow 8$

12. 

$1 + 317 + 248 - x = 450$

$x = 566 - 450 = 116$

13. $\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \cdots$

$= \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots = \frac{1}{2}$

$+ \frac{1}{9} + \frac{1}{27} + \cdots = \frac{1}{6}$

$+ \frac{1}{27} + \cdots = \frac{1}{18}$

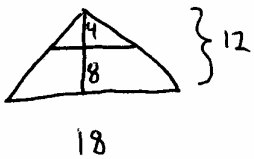
$+ \cdots = \frac{3}{4}$

14. $d = \frac{32-12}{17-12} = \frac{20}{5} = 4$

$\Rightarrow a = 12 - 11 \cdot 4 = -32$

$S_{20} = \frac{(a + (a+19d))}{2} 10$

$= (-64+76) \cdot 10 = \boxed{120}$

15. 

$\frac{4}{12} = \frac{b_2}{18} \Rightarrow b_2 = 6$

$A = \frac{1}{2} \cdot 4 \cdot 6 = \boxed{12}$

16. $y' = 3x^2 - 8x + 4 = 0$

$x = \frac{8 \pm \sqrt{64-48}}{6} = \frac{8 \pm 4}{6} = 2, \frac{2}{3}$

$y'' = 6x - 8$

$y''(2) = 4 > 0$ ✓

$y''(\frac{2}{3}) = -4 < 0$ ✗

$y(2) = 8 - 16 + 8 - 7 = -7$ $\boxed{(2, -7)}$

17. $x^2 - 2xy + y^2 = 100$

$+4xy \qquad +4 \cdot 8$

$x^2 + 2xy + y^2 = 132$

$x + y = \pm\sqrt{132} = \pm 2\sqrt{33}$

$x^2 - y^2 = (x+y)(x-y) = \pm 2\sqrt{33} \cdot 10 = \boxed{\pm 20\sqrt{33}}$

18. $\sqrt{12 \cdot 288} = 4\sqrt{3 \cdot 72}$

$= 24\sqrt{3 \cdot 2} = \boxed{24\sqrt{6}}$

19. $5 \cdot 36_{10} + 3 \cdot 6_{10} + 3_{10} = 201$

$= 3 \cdot 64 + 0 \cdot 16 + 2 \cdot 4 + 1$

$\boxed{3021_4}$

20. $r^2 + s^2 + t^2 = (r+s+t)^2 - 2(rs + st + rt)$

$= \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right)$

$= \frac{16}{9} - 2(-1) = \boxed{\frac{34}{9}}$

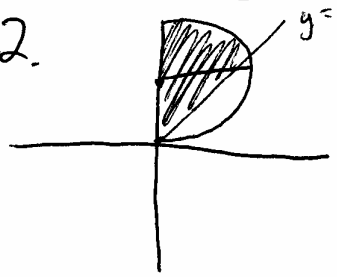
21. $5760 = 2 \cdot 5 \cdot 4 \cdot 144$

$= 2^7 \cdot 3^2 \cdot 5^1 \Rightarrow b = 1$

$c = 2$

$a = \frac{5}{2}$

Sum $= \boxed{\frac{11}{2}}$

22. 

$r < \sin\theta$

$r < \frac{y}{r}$

$r^2 < y$

$x^2 + y^2 - y < 0$

$x^2 + (y - \frac{1}{2})^2 < \frac{1}{4}$

$A = \frac{1}{4}\left(\frac{1}{2}\right)^2 \pi + \frac{1}{2}\left(\frac{1}{2} \cdot \frac{1}{2}\right) = \boxed{\frac{\pi + 2}{16}}$

23. $A = \frac{1}{2} P r = \sqrt{s(s-a)(s-b)(s-c)}$

$\frac{1}{2} \cdot 22 \cdot r = \sqrt{11 \cdot 2 \cdot 4 \cdot 5}$

$r = \boxed{\frac{2\sqrt{110}}{11}}$

24. $2\sum_{1}^{50} - 6\sum_{1}^{16}$

$2\cdot\frac{50\cdot51}{2} - 6^{3}\cdot\frac{16\cdot17}{2}$

$50\cdot51 - 16\cdot51$

$34\cdot51 = (1734)$

25. 115 3
124 6 $\Rightarrow \frac{15}{216} = (\frac{5}{72})$
133 3
223 3

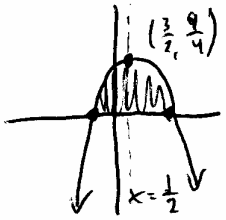
26. $30 = 2\cdot3\cdot5$
$36 = 2^2\cdot3^2$
$50 = 2\cdot5^2$
$LCM = 2^2\cdot3^2\cdot5^2 = 900 \Rightarrow (1801)$

27. $a^2n + b^2m = t^2c + mcn$
$2^2\cdot2 + 5^2\cdot3 = x^2\cdot5 + 3\cdot5\cdot2$
$83 = 5x^2 + 30$
$\sqrt{\frac{53}{5}} = x = (\frac{\sqrt{265}}{5})$

28. There are at least two people outside the group of 5 who haven't shaken hands with everyone (because the 5 need at least two people to not shake hands with). $2+5=7 \Rightarrow 24-7 = (17)$

29. $3n+1$
$5m+1 \Rightarrow 15_0+1$
$7_p \Rightarrow (91)$

30. $4=4 = 2\cdot2$
$p^3 \quad p^1q^1$
$2^3=8 \quad 2\cdot3=6$
$3^3=\cancel{27} \quad 2\cdot5=10$
$2\cdot7=\cancel{14}$ Sum $= (24)$

31. 

$\int_{-1}^{2} -x^2+x+2$

$-\frac{1}{3}x^3+\frac{1}{2}x^2+2x\Big|_{-1}^{2}$

$-\frac{1}{3}(8-(-1))+\frac{1}{2}(4-1)+2(2-(-1))$

$-3+\frac{3}{2}+6 = (\frac{9}{2})$

32. $\left(\frac{3}{4}(100)^2 + \frac{1}{4}(50^2) + \frac{1}{4}(40)^2\right)\pi$

$(3\cdot50^2 + 25^2 + 20^2)\pi$

$(7500+625+400)\pi = (8525\pi)$

33. 0 0 0 0 x x x x x

$5x = 9\cdot35 = 315$

$x = (63)$

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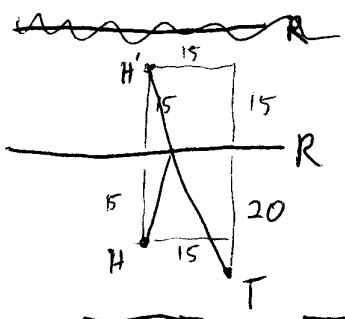
34. 

$d = \sqrt{35^2+15^2} = 5\sqrt{7^2+3^2} = 5\sqrt{58}$

35. $AB = \begin{bmatrix} 3 & -18 \\ -2 & -12 \end{bmatrix}$ $BA = \begin{bmatrix} 9 & -5 \\ 8 & -8 \end{bmatrix}$

$AB - BA = \begin{bmatrix} -6 & -8 \\ -10 & -6 \end{bmatrix}$

36. HHTXX 4

THHTX 2

XTHHT 2

XXTHH 3 (can't be HHTHH)

$\frac{11}{32}$

37. $\frac{\sqrt{3}}{2} + 1 + \frac{\sqrt{3}}{2} + -2 + 2 + 0$

38. $\binom{6}{3} = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2} = 20$

39. $\frac{13-5i}{22+7i} \cdot \frac{22-7i}{22-7i} = \frac{286-110i-91-35}{484+49}$

$= \frac{251-201i}{533}$

40. $2\int_0^1 xe^x$ $u = x$ $dv = e^x$ $du = dx$ $v = e^x$

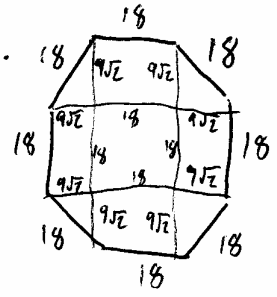
$2\left[xe^x\Big|_0^1 - \int_0^1 e^x\right]$

$2\left[(x-1)e^x\right]_0^1 = 2[-1] = 2$

41. They coincide 11 times in 12 hours

22 times a day

There's a 30° angle before & after each coincidence $\Rightarrow 44$

42. 

$A = 18^2 + 4 \cdot \frac{1}{2}(9\sqrt{2})^2 + 4 \cdot 18 \cdot 9\sqrt{2}$

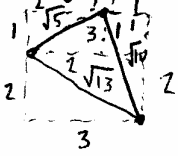
$= 324 + 324 + 648\sqrt{2}$

$= 648 + 648\sqrt{2}$

43. $\sqrt{13} = \sqrt{9+4} \Rightarrow 3, 2$

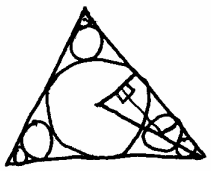
$\sqrt{5} = \sqrt{4+1} \Rightarrow 2, 1$

$\sqrt{10} = \sqrt{9+1} \Rightarrow 1, 3$



$A = 3 \cdot 3 - \frac{1}{2}(2 \cdot 1 + 3 \cdot 1 + 3 \cdot 2)$

$= 9 - \frac{11}{2} = \frac{7}{2}$

44. 

$r_1 = 5$

$r_2 + r_1 = 2(r_1 - r_2)$

$3r_2 = r_1$

$r_2 = \frac{r_1}{3} = \frac{5}{3}$

$C_1 = 2\pi r_1 = 10\pi$

$C_{2\to\infty} = \frac{2\pi r_2}{1-\frac{1}{3}} = 5\pi$

$3 \cdot C_{2\to\infty} = 15\pi$ = **25π**

45. $\frac{150}{11} = 13$ r 7 $\quad 13+1 =$ **14**

$\frac{13+7}{11} = 1$ r 9

$\frac{1+9}{11} = 0$ r 10

46. We should put the smallest number in each pigeonhole as possible $\Rightarrow 1+2+3+\dots = \frac{n(n+1)}{2} < 200$

$n^2+n < 400 \Rightarrow n =$ **19**

47. $\binom{10}{6} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2} = 210$ total ways

5D 1N $= \binom{5}{1} = 5$

5D 1P $= \binom{2}{1} = 2$ $\Rightarrow$ 125 good ways

4D XX $= \binom{5}{2}\binom{5}{1}\binom{4}{1} = 10\cdot 5 = 50$

3D 3N $= \binom{5}{3} = 10$

3D 2N 1P $= \binom{5}{3}\binom{3}{2}\binom{2}{1} = 60$

$\frac{125}{210} =$ **$\frac{25}{42}$**

48. 3W 5W
4B 2B

$\frac{3}{7}$W $\to \frac{6}{8}$W

$\frac{4}{7}$B $\to \frac{5}{8}$W

$\frac{20}{20+18} = \frac{20}{38} =$ **$\frac{10}{19}$**

49. $2, 5, \frac{16}{5}, \frac{107}{25}$

$+3 \quad -\frac{9}{5} \quad +\frac{27}{25}$

$\lim = 2 + \frac{3}{1-\frac{3}{5}} = 2 + \frac{15}{8} =$ **$\frac{31}{8}$**

50. $\vec{A} = [5, 1, -3]$

$\vec{B} = [4, -3, -2]$

$\vec{A} \times \vec{B} = [-11, -2, -19]$

$11x + 2y + 19z = 35$

51. $\int_0^{\pi} \sin^2(3x) = \int_0^{\pi} \frac{1-\cos(6x)}{2}$

$= \frac{1}{2}x - \frac{1}{12}\sin(6x)\Big|_0^{\pi}$

$= \frac{1}{2}[(\pi - 0) - \frac{1}{12}(0-0)] =$ **$\frac{\pi}{2}$**

52. $-\sqrt{3^2+2^2} =$ **$-\sqrt{13}$** (for 2 sinusoids 90° out of phase)

OR $y = 3\sin\theta - 2\cos\theta$

$y' = 3\cos\theta + 2\sin\theta = 0$

$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{3}{2}$

$\sin\theta = -\frac{3}{2}\cos\theta = \sqrt{1-\cos^2\theta}$

$\frac{4}{9}\cos^2\theta = 1 - \cos^2\theta$

$\cos\theta = \sqrt{\frac{9}{13}} \quad \sin\theta = -\sqrt{\frac{4}{13}}$

53. $\left(2\left(34\cdot\frac{3}{2}+\frac{1}{2}\right)-\frac{1}{3}\right)\frac{3}{2}$

$\left(103-\frac{1}{3}\right)\frac{3}{2}=154$

54. $P(\text{not full after 4}) = NNNN = \left(\frac{3}{4}\right)^4$

$+4\,NNNE = +4\left(\frac{3}{4}\right)^3\cdot\frac{1}{4}$

$= \frac{189}{256} \times \frac{1}{4} = \frac{189}{1024}$

55.

1 4 10	~~339~~	$\Rightarrow \frac{10}{\binom{10}{3}}$
1 5 9	348	
1 6 8	357	$= \frac{10\cdot 3\cdot 2}{10\cdot 9\cdot 8} = \frac{1}{12}$
2 3 10	456	
2 4 9		
2 5 8		
2 6 7		

56. She arrives home 20 minutes early $\Rightarrow$ the car would have driven 20 minutes longer had it not met her $\Rightarrow$ 10 minutes before 5:00 they met $\Rightarrow$ she walked 50 minutes

57. $12^2 = 144$

The differences b/t consecutive squares are consecutive odd numbers, so we'd like an odd number of terms centered on an odd number (impossible for 144) or an even centered on an even.

$144 = 2\cdot 2\cdot 36 = 2\cdot 2\cdot 2^2\cdot 3^2$

2·72	71, 73 $\Rightarrow$ 35
4·36	33, 35, 37, 39 $\Rightarrow$ 16
6·24	19 $\Rightarrow$ 9
8·18	11 $\Rightarrow$ 5
12·12	1 $\Rightarrow$ ~~0~~

sum = 65

58. A returns every 600s

B 1100s

they meet at start every LCM = 3000s

they meet some where $5t+3t=3000$

$t = \frac{3000}{8}$

$\Rightarrow 8+1=9$

59. $C = np = (n+3)(p-1) = (n-2)(p+1)$

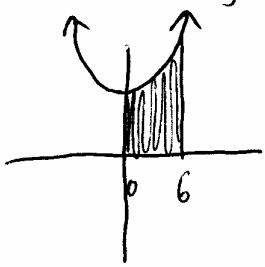
$3p - n = 3$

$-2p + n = 2$

$p = 5 \Rightarrow n = 12 \Rightarrow C = 60$

$\frac{60}{5} = 12$

60.



$\int_0^6 2\pi x(x^2+4)$

$2\pi\int_0^6 x^3+4x$

$2\pi\left[\frac{1}{4}x^4+2x^2\right]_0^6$

$2\pi(324+72)$

792π