Mu Alpha Theta National Convention: Denver, 2001

State Bowl – Mu Division

1. What is the shape of the graph of the equation $4x^2 - 8x - 9y^2 - 108y = 113$?

2. What is the harmonic mean of 3 and 5?

3. Evaluate: $\log_8 81 \cdot \log_5 64^{\log_9 125}$

4. A particle's position s (measured in meters from the origin of a number line) at time t (measured in seconds) is $s(t) = 3t^3 - \frac{18}{t}$. What is the particle's velocity, in meters per second, at $t = 3$ seconds?

5. Evaluate: $\int_0^2 \left(x^3 - 3x^2 + 3x - 2\right)dx$

6. A triangle has a perimeter of 128, and its inscribed circle has an area of 8π. What is the area of the triangle?

7. Determine the sum of all values of $\theta \left(0 \le \theta < 2\pi\right)$ for which $\cos^2(\theta) = \frac{3}{5}$.

8. Given that $\log 4 = a$, express $\log 20$ in terms of a.

9. Determine the product of the eigenvalues of the matrix $\begin{bmatrix} 1 & 4 \\ -2 & 8 \end{bmatrix}$.

10. Determine the sum of all possible values of h for which $\frac{2h-1}{h+3} = \frac{h+4}{3h-2}$.

11. A bag contains red and blue marbles. When two marbles are drawn, the probability that they are both red is equal to the probability they are both blue. The probability that one of each color is drawn is $\frac{4}{7}$. How many marbles are in the bag?

12. At the Clymer Academy of Math and Science there are 450 students. 317 students take Math, 248 take Science, and one takes neither. How many students take both?

13. Evaluate: $\frac{1}{3}+\frac{2}{9}+\frac{3}{27}+\frac{4}{81}+\cdots+\frac{n}{3^n}+\cdots$

14. The twelfth term of an arithmetic sequence is 12, and the seventeenth term is 32. What is the sum of the first twenty terms of the sequence?

15. A triangle with height 12 centimeters and base 18 centimeters (this height and this base are perpendicular to one another) is cut by a line parallel to the base and eight centimeters from it, dividing the original triangle into a smaller triangle and a trapezoid. What is the area of the smaller triangle, in square centimeters?

16. Express as an ordered pair the x and y coordinates of the local minimum of the equation $y = x^3 - 4x^2 + 4x - 7$.

17. If $x - y = 10$ and $xy = 8$, determine all possible values of $x^2 - y^2$.

18. What is the geometric mean of 12 and 288?

19. Express 533_6 in base 4.

20. Determine the sum of the squares of the roots of $3x^3 - 4x^2 - 3x + 2 = 0$.

21. The number 5760 can be expressed as $4^a \cdot 5^b \cdot 6^c$, where *a*, *b*, and *c* are rational numbers. Evaluate $a+b+c$.

22. What is the area of the region in the first quadrant satisfying the rectangular condition $y > x$, and the polar condition $r < \sin\theta$?

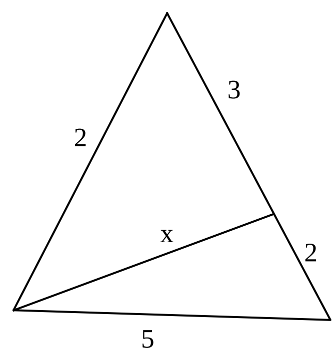
23. What is the length of the radius of the inscribed circle of a triangle with sides of 6, 7 and 9?

24. Determine the sum of all the natural numbers from 1 to 100 inclusive that are divisible by two but not three.

25. Three dice are rolled. What is the probability that they sum to seven?

26. What is the smallest natural number greater than 1000 that leaves the same non-zero remainder when divided by 30, 36, and 50?

27. What is the length of *x* in the triangle?



28. In a room containing 24 people, there is a group of five people who have not shaken hands with everyone else, but have shaken hands with each of the other four, and there is no person with whom the five have all failed to shake hands. What is the maximum number of people who could have shaken hands with everyone else?

29. While the marching band was practicing for its big performance, they marched in rows of three, but this left poor Kathy all alone in the last row. The director told them to march in rows of five to correct this, but Kathy was still alone in the back. Exasperated, Kathy finally told him the number of people in the band was divisible by seven. What is the fewest number of band members possible?

30. Find the sum of the three smallest natural numbers with exactly four positive integral factors.

31. What is the area of the region satisfying the constraints $y < (x+1)(2-x)$ and $y > 0$?

32. A goat is tethered to an external corner of a 50 by 60-meter rectangular barn with a rope 100 meters long. What is the area of the region the goat can freely graze, in square meters?

33. What is the maximum possible value of the median of a nine-member data set whose elements are whole numbers from zero to 100 inclusive, the arithmetic mean of which is 35?

34. A pioneer is headed home from town, but must stop at the river along the way to get water. If the river runs east-west twenty kilometers north of the town, and the pioneer's home is fifteen kilometers west and five kilometers north of the town, what is the shortest distance, in kilometers, the pioneer can travel?

35. If $A = \begin{bmatrix} 3 & 1 \\ -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ 0 & -4 \end{bmatrix}$, evaluate $AB - BA$.

36. A coin is to be flipped five times. What is the probability that within the five flips, the longest sequence of heads will be exactly two heads in length?

37. Evaluate: $\sin\left(\frac{\pi}{3}\right)+\tan\left(\frac{\pi}{4}\right)+\cos\left(\frac{7\pi}{6}\right)+\sec\left(\frac{4\pi}{3}\right)+\csc\left(\frac{\pi}{6}\right)+\cot\left(\frac{3\pi}{2}\right)$

38. What is the maximum number of triangles that can be formed by six lines in a plane?

39. Simplify: $\frac{13-5i}{22+7i}$

40. Evaluate: $\int_0^1 2xe^x\,dx$

41. How many times each day does the smaller angle between the minute hand and the hour hand of a clock equal 30°?

42. What is the area of a regular octagon with perimeter 144?

43. Determine the area of a triangle with sides of $\sqrt{13}$, $\sqrt{5}$, and $\sqrt{10}$.

44. A circle of radius five is inscribed in an equilateral triangle. For each vertex, a circle is inscribed tangent to the two sides that meet at that vertex, and also tangent to the circular arc closest to that vertex. If this process is continued indefinitely, what is the sum of the circumferences of all of the circles?

45. A Coca-Cola addict knows that when someone "empties" a can of Coke by drinking it, it is not really empty. So, he has built a contraption that will allow him to take eleven "empty" cans and produce one full can of Coke as a result (truly emptying the eleven cans in the process). If he throws a party and finds 150 "empty" cans on his counter the next morning, how many full cans could he drink?

46. What is the largest number of pigeonholes which 200 pigeons can occupy, given that there must be at least one pigeon in each hole and that no two holes can contain the same number of pigeons?

47. A box contains two pennies, three nickels, and five dimes. What is the probability that when 6 coins are drawn their total value is greater than forty cents?

48. You have two bags of pebbles. Bag A contains three white and four black pebbles, while bag B contains five white and two black pebbles. A pebble is taken from bag A and placed in bag B. If a pebble then drawn from bag B is white, what is the probability that the pebble transferred from bag A was black?

49. A sequence is defined as:

$$F_1 = 2$$
$$F_2 = 5$$
$$F_n = \frac{2F_{n-1} + 3F_{n-2}}{5} \text{ for } n > 2$$

What is the limit of this sequence as n becomes large?

50. What is the equation of the plane through the points $(-2, 0, 3)$, $(3, 1, 0)$, and $(2, -3, 1)$?

51. Evaluate: $\int_0^{\pi} \sin^2(3x)dx$

52. What is the minimum value of the expression $3\sin\theta - 2\cos\theta$, where θ is a real number?

53. A woman decides to get rid of her goldfish. To a friend of hers she gives a third of her fish, minus one-third of a fish. Then she sells half of the remainder plus one-half to a pet shop. She then sells one-third of the remainder to another pet shop, leaving her with 34 fish she was unable to find a home for. How many fish did she start with?

54. A spider eats two flies a day. Until he has done so, he has a 25% chance of eating any fly that passes by, after which he will let any passing fly pass unmolested. What is the probability that the fifth fly to attempt to pass will be eaten?

55. Three distinct numbers are chosen from the first ten natural numbers. What is the probability that their sum is 15?

56. A woman typically takes the train home from work, arriving at the station at 5 o’clock, just as her husband arrives in the car to pick her up. One day the woman is able to leave work early and arrives at the station at 4 o’clock. Rather than calling her husband, she immediately begins walking along the route she knows he will take, meeting him en route and arriving home twenty minutes earlier than she usually does. For how many minutes did the woman walk?

57. One leg of a Pythagorean triangle is 12. Determine the sum of all possible values of the other leg.

58. Two people start walking in opposite directions from the same point on a circular track three kilometers in circumference. If their walking speeds are three and five meters per second, how many times will they be at the same point on the track by the time they next meet at their starting positions, including both their initial and final colocations?

59. A math team decides to order five extra-large pizzas. If there were three more people on the team, the cost would have been one dollar less per person, though if there'd been two fewer people, the cost would have been one dollar more per person. What is the price, in dollars, of an extra-large pizza?

60. What is the volume of the solid formed by the rotation of the region satisfying the conditions $y < 4 + x^2$, $|x - 3| < 3$, and $y > 0$ about the y-axis?