Alpha Sequences & Series Test — MAΘ National Convention 2025

Important Instructions for this Test:
Good luck, have fun, and as always: "NOTA" stands for "None of These Answers is correct."

1. The first term of a sequence is 23. If each term, after the first term, is 4 more than the previous term, then what is the 10th term of the sequence?

A: 55 B: 59 C: 63 D: 67 E: NOTA

2. What is the value of $\sum_{n=1}^{5}(9n-7)$?

A: 10 B: 40 C: 100 D: 128 E: NOTA

3. In a sequence, each term, after the first term, is 5 more than the previous term. If the first and last terms are 30 and 105, respectively, then how many terms are there in the sequence?

A: 14 B: 15 C: 16 D: 17 E: NOTA

4. What is the value of $\sum_{n=0}^{5}2(n+3)^2$?

A: 90 B: 146 C: 270 D: 398 E: NOTA

5. What is the sum of the first 800 positive integers?

A: 320,000 B: 320,400 C: 640,000 D: 640,800 E: NOTA

6. What is the sum of the first 100 positive integer perfect cubes, minus the sum of the first 5,050 positive odd integers?

A: 100 B: 50 C: 0 D: −50 E: NOTA

7. What is the value of $\dfrac{\sum_{n=200}^{299}(2n+1)}{\sum_{n=100}^{199}(2n+1)}$?

A: $\frac{5}{3}$ B: $\frac{3}{2}$ C: $\frac{499}{299}$ D: $\frac{249}{149}$ E: NOTA

8. What is the sum of the first 60 positive integer perfect squares?

A: 3,600 B: 73,810 C: 216,000 D: 3,348,900 E: NOTA

9. What is the coefficient of the x^5 term of the binomial expansion of $(x-1)^9$?

A: −126 B: −45 C: 45 D: 126 E: NOTA

10. Which algebraic expression represents the *n*th term of a geometric sequence with 4 as the first term and 10 as the second term?

A: $4\left(\frac{5}{2}\right)^{n-1}$ B: $4\left(\frac{5}{2}\right)^{n}$ C: $4+6(n-1)$ D: $4+6n$ E: NOTA

11. What is the sum of an infinite geometric series with first and second terms 4 and $-\frac{1}{2}$, respectively?

A: $\frac{8}{3}$ B: $\frac{7}{2}$ C: $\frac{32}{9}$ D: $\frac{32}{7}$ E: NOTA

12. If the terms of the binomial expansion of $(2x+3)^8$ are written in order of decreasing degree, then what is the coefficient on the 6th term of this expansion?

A: 108,864 B: 81,648 C: 48,384 D: 16,128 E: NOTA

13. In an infinite geometric series, the first two terms are each less than 32. If the sum of the first two terms is 32, and the sum of the entire infinite series is 50, then what is the common ratio of the terms in this series?

A: $\frac{9}{25}$ B: $\frac{9}{16}$ C: $\frac{3}{5}$ D: $\frac{3}{4}$ E: NOTA

14. In an infinite geometric series, the first term is 1. If the sum of the entire infinite series is $\frac{3}{2}$, then what is the sum of the cubes of all the terms in this series?

A: $\frac{27}{8}$ B: $\frac{9}{4}$ C: $\frac{9}{8}$ D: $\frac{27}{26}$ E: NOTA

15. If a is a real number such that $\sum_{n=1}^{\infty} a(0.6)^n = 54$, then what is the value of $\sum_{n=1}^{\infty} a(0.8)^n$?

A: 64 B: 72 C: 81 D: 108 E: NOTA

16. In general, for all real numbers x, $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ where x is in radians.

If $0 \le a \le \frac{\pi}{2}$ and $\sum_{n=0}^{\infty} \frac{(-1)^n a^{2n+1}}{(2n+1)!} = 0.1$, then what is the value of $\sum_{n=0}^{\infty} \frac{(-4)^n a^{2n+1}}{(2n+1)!}$?

A: $\frac{3\sqrt{11}}{100}$ B: $\frac{3\sqrt{11}}{50}$ C: 0.2 D: 0.4 E: NOTA

17. Irina just opened up a new empty savings account that pays 3% annual interest, compounded annually. She plans to make equal annual deposits, with the first deposit immediately and the last deposit exactly 20 years from now. Assuming no other transactions on this account, which expression shows the amount (in dollars) needed for each deposit, in order for Irina to have an account balance of exactly $100,000 exactly 21 years from now?

A: $\dfrac{3{,}000}{(1.03)^{22}-1.03}$ C: $\dfrac{3{,}000}{(1.03)^{21}-1.03}$ E: NOTA

B: $\dfrac{3{,}000}{(1.03)^{21}-1}$ D: $\dfrac{3{,}000}{(1.03)^{20}-1}$

18. Let X represent a nonnegative integer valued random variable such that $P(X=n)=p^n(1-p)$ for all nonnegative integers n, where p is a constant in the interval $(0, 1)$. For what value of p does X have expected value equal to 4?

A: 0.2 B: 0.25 C: 0.75 D: 0.8 E: NOTA

19. What is the value of $\displaystyle\sum_{n=1}^{\infty}\left(\frac{0.5n+2}{0.5n+1}-\frac{0.5n+3}{0.5n+2}\right)$?

A: $\dfrac{2}{3}$ B: $\dfrac{5}{3}$ C: $\dfrac{13}{6}$ D: $\dfrac{19}{6}$ E: NOTA

20. What is the value of $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\left[3\left(\frac{k}{n}\right)^3+\frac{k}{n}\right]$?

A: 0 B: $\dfrac{5}{4}$ C: 2 D: $\dfrac{5}{2}$ E: NOTA

21. In the game Red-Blue Hackenbush, the value of an infinite stack of sticks, containing infinitely sticks of each color, is the total value of the blue sticks minus the total value of the red sticks. Starting from the bottom, each stick is worth 1 before the first color change. At the first color change and upward, each stick is worth half as much as the stick immediately below it. Consider the following stack, in order from bottom to top: 1 blue, 3 reds, 3 blues, 3 reds, 3 blues, 3 reds, 3 blues, with the last 6 sticks repeating forever. What is the value of the stack?

A: $\dfrac{1}{9}$ B: $\dfrac{8}{63}$ C: $\dfrac{1}{7}$ D: $\dfrac{2}{9}$ E: NOTA

22. What is the value of $\displaystyle\sum_{n=3}^{\infty}\frac{n}{n^4-2n^2+1}$?

A: $\dfrac{1}{16}$ B: $\dfrac{13}{144}$ C: $\dfrac{1}{4}$ D: $\dfrac{13}{36}$ E: NOTA

23. What is the value of $\prod_{n=0}^{\infty} \frac{3 \cdot 2^n + 1}{3 \cdot 2^n + 2}$?

A: 0 B: $\frac{1}{3}$ C: $\frac{1}{2}$ D: $\frac{3}{5}$ E: NOTA

24. Mike has a coin that comes up heads with probability *p*, and Nathan has a fair coin. They take turns tossing their own coin, until one player obtains a head and wins. If Mike goes first and has probability $\frac{4}{7}$ of winning the game, then what is the value of *p*?

A: $\frac{4}{11}$ B: $\frac{2}{5}$ C: $\frac{1}{2}$ D: $\frac{4}{7}$ E: NOTA

25. Joyce and Lynn take turns rolling the same pair of standard fair 6-sided dice, until one of them rolls a sum of at least 8 on the two dice and wins. If Joyce goes first, then what is the probability that Joyce wins?

A: $\frac{5}{12}$ B: $\frac{36}{67}$ C: $\frac{12}{19}$ D: $\frac{5}{7}$ E: NOTA

26. Alice, Beth, and Cherise, in this order, take turns rolling the same standard fair 6-sided die, until one of them rolls a 6 and wins. If Alice goes first, then what is the probability that Beth wins?

A: $\frac{1}{6}$ B: $\frac{1}{3}$ C: $\frac{36}{91}$ D: $\frac{30}{91}$ E: NOTA

27. Alex, Boris, and Carlos, in this order, take turns flipping the same coin with nonzero probability *p* of showing heads, until one of them obtains a head and wins. If Alex goes first and wins the game with probability $\frac{1}{2}$, then what is the value of *p*?

A: $\frac{1}{3}$ B: $\frac{3-\sqrt{5}}{2}$ C: $\sqrt{2}-1$ D: $\frac{1}{2}$ E: NOTA

28. Two volleyball teams, A and B, play until one team wins two more sets than the other team and so wins the match. Team A has constant probability *p* of winning any given set, the results of the sets are independent, and a winner is decided in each set. If team A has probability $\frac{25}{169}$ of winning the match, then what is the value of *p*?

A: $\frac{5}{12}$ B: $\frac{5}{13}$ C: $\frac{5}{17}$ D: $\frac{5}{18}$ E: NOTA

29. If *x* is a real number such that $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = 0.2$, then what is the value of $\sum_{n=0}^{\infty} \frac{(-9)^n x^{2n}}{(2n)!}$?

A: 0.6 B: 0.568 C: –0.568 D: –0.6 E: NOTA

30. What is the value of $\sum_{n=0}^{\infty} \frac{(\ln 2.5)^{2n+1}}{(2n+1)!}$?

A: 1.05 B: 1.45 C: 2.1 D: 2.5 E: NOTA