Answer Key:

1. B
2. D
3. D
4. B
5. A
6. B
7. C
8. A
9. B
10. A
11. B
12. D
13. D
14. B
15. B
16. C
17. B
18. A
19. A
20. A
21. E
22. D
23. C
24. A
25. D
26. D
27. A
28. C
29. C
30. B

Solutions:

1. $\boxed{\textbf{B}}$: $a = \frac{3}{4}, b = \frac{13}{6}, c = \frac{40}{13}$, multiplying them together results in 5.

2. $\boxed{\textbf{D}}$: Rearranging into polynomial form results in the equation $(\log_{10} x)^2 + \log_{10} x - 6 = 0$, resulting in the roots $(\log_{10} x + 3)(\log_{10} x - 2)$, meaning $x = 0.001$ or 100

3. $\boxed{\textbf{D}}$: First, solving for $f(\frac{1}{x})$ gives the expression $\sqrt{1 - (\log_{\frac{1}{x}} 10)^2} = \sqrt{1 - (-\log_x 10)^2} = \sqrt{1 - (\log_x 10)^2}$, which means $0 < x \leq \frac{1}{10}$ or $10 \leq x < \infty$. For $f(-\frac{1}{x})$, the domain will simply be $(-\infty, -10) \cup [-\frac{1}{10}, 0)$

4. $\boxed{\textbf{B}}$: Using exponent rules, this expression is simplified to $e^{a^b} = e^{ab}$, or $a^b = ab$. This can be simplified to $a^{b-1} = b$, and then $a = \sqrt[b-1]{b}$

5. $\boxed{\textbf{A}}$: The real part of e^{ix} is $\cos x$, and $\frac{e^{2ix}}{\sqrt{e^{\pi i}}}$ can be simplified to $\frac{\cos 2x + i \sin 2x}{\sqrt{-1}}$, the real part of which is $\sin 2x$. Solving for the roots of $\cos x = \sin 2x$ gives the answers $\frac{\pi}{2}$, $\frac{3\pi}{2}$, $\frac{\pi}{6}$, and $\frac{5\pi}{6}$.

6. $\boxed{\textbf{B}}$: $e^{-\ln(x^2+1)}$ is even, since x^2 is even. However, since $\sin x$ is odd, the entire function is odd, since an odd function multiplied by an even function is odd.

7. $\boxed{\textbf{C}}$: Rearranging into quadratic form gives the equation $11 * 11^{2x} - 121 * 11^x + 330$, or $11(11^x - 5)(11^x - 6)$, which means $x = \log_{11} 5 \text{ or} \log_{11} 6$

8. $\boxed{\textbf{A}}$: $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, so $\tanh -x = \frac{e^{-x} - e^x}{e^x + e^{-x}}$. $\sinh x + \cosh x = e^x$, so $\frac{\tanh(-x)}{\sinh x + \cosh x} = \frac{e^{-2x} - 1}{e^x + e^{-x}} = \frac{1 - e^{2x}}{e^x + e^{3x}}$

9. $\boxed{\textbf{B}}$: The expression can be simplified to 3^{-x^2+6x-3}. $-x^2 + 6x - 3 = -(x-3)^2 + 6$ meaning the maximum value of the quadratic is 6, and the maximum value of the expression is 3^6.

10. $\boxed{\textbf{A}}$: Logarithms can take positive, non-zero arguments. $x^2 + 2x - 3$ is positive for $(-\infty, -3) \cup (1, \infty)$. $x + 3$ is positive for $(-3, \infty)$. $x - 3$ is positive for $(3, \infty)$. The intersection of all of these is $(3, \infty)$. However, since the denominator of the fraction cannot be 0, $x \neq 4$. Thus, the answer is $(3, 4) \cup (4, \infty)$

11. $\boxed{\textbf{B}}$: A point is rational if y is a power of 3 and x is a power of 2. There are 7 powera of 3 under 1000 and 10 powers of 2 under 1000, meaning there are 70 possible points.

12. $\boxed{\textbf{D}}$: The taylor series of e^x is $e^x = \frac{x}{0!} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + ...$, which means the approximation for $e^x + e^{-x} = 2 + x^2 + \frac{x^4}{12}$. This means 2, $x^2 + 2$, and $2 + x^2 + \frac{x^4}{12}$ count as approximation, but $\frac{x^3}{3} + x^2 + x + 2$ does not.

13. $\boxed{\textbf{D}}$: $\frac{ab}{a+b} = (\frac{1}{a} + \frac{1}{b})^{-1}$, so for $a = \frac{\log_{10} x}{\ln 10x}$ and $b = \frac{1}{\ln 10 * \log_x \frac{x^2}{10}}$, $(\frac{\ln(10x)}{\log_{10} x} + \ln 10 * \log_x \frac{x^2}{10})^{-1} = (\frac{\ln(10x) + \log_{10} x * \ln 10 * \log_x \frac{x^2}{10}}{\log_{10} x})^{-1} = (\frac{\ln(10x) + 2\log_{10} x * \ln 10 - \ln 10}{\log_{10} x})^{-1} = (\frac{\ln(x) + 2\log_{10} x * \ln 10}{\log_{10} x})^{-1} = (\ln 10 + 2\ln 10)^{-1} = \frac{1}{3 \ln 10} = \frac{1}{\ln 1000}$

14. $\boxed{\textbf{B}}$: Logarithms only take positive arguments. $x^2 - 1$ is positive for $(-\infty, 1) \cup (1, \infty)$. $\frac{x^2+4x-5}{x+1}$ is positive for $(-5, -1) \cup (1, \infty)$. $\frac{x^2 + \frac{7}{2}x - \frac{15}{2}}{x - \frac{3}{2}}$ simplifies to $x + 5$, which is positive for $(-5, \infty)$. The intersection of these is $(-5, -1) \cup (1, \infty)$. If the expression is simplified to $\ln(\frac{(x+1)^2(x-1)(x-\frac{3}{2})(x+5)}{(x+5)(x-1)(x-\frac{3}{2})})$, it's clear that the only hole in the domain is $\{\frac{3}{2}\}$

15. **B** : $2 = e^a$, so $(e^{2a})^x = e^{a+1}$, so $2ax = a + 1$, and $x = \frac{a+1}{2a}$

16. **C** : $\lim_{x\to\infty}(1 + \frac{n}{x})^x = e^n$, therefore $\lim_{x\to\infty}(1 + \frac{i}{x})^{\pi x} = e^{\pi i} = -1$

17. **B** : $\sqrt{x^2 + x} > 1$ in order to be positive in the square root, and avoid a zero in the denominator. Solving for the bounds of this inequality gives the polynomial $x^2 + x - 1 = 0$, so the roots must be $\frac{-1\pm\sqrt{5}}{2}$, and the function exists for $(-\infty, \frac{-1-\sqrt{5}}{2}) \cup (\frac{-1+\sqrt{5}}{2}, \infty)$

18. **A** : If V is the volume of a single violin, and F is the volume of a single flute, then $10\log_{10}(\frac{8V}{F}) = 15$, which means $10^{\frac{3}{2}} \approx 32 = \frac{8V}{F}$, and $F = \frac{V}{4}$. For the volume to be equal, $10\log_{10}(\frac{aV}{20F})$ must equal 0 for a violins in the orchestra. This means $\frac{aV}{20F} = 1$, or $\frac{a}{5} = 1$, meaning $a = 5$.

19. **A** : The maximum amplitude of the voltage of the signal is 22.5, and the minimum is 5. This means the difference in decibels is $10\log_{10}(\frac{\frac{22.5^2}{R}}{\frac{5^2}{R}}) = 20\log_{10}(\frac{45}{10}) = 20\log_{10}(45) - 20 = 10\log_{10}(2025) - 20 = 33.06 - 20 = 13.06$

20. **A** : $\log_{(x-m)}(x - n) = \frac{\ln(x-n)}{\ln(x-m)}$, meaning the function has a vertical asymptote for $x = m + 1$. The domain of $\ln(x - n)$ is (n, ∞), and$\ln(x - m)$ is (m, ∞), meaning both n and m must be lower than $m + 1$ for the vertical asymptote to exist. Lastly, n cannot equal m as the logarithm would cancel out, meaning the answer is $(-\infty, m) \cup (m, m + 1)$.

21. **E** : The holes of $\frac{1}{\cos(\ln x)}$ occur where $\cos(\ln x) = 0$, or where $\ln x = k\pi + \frac{\pi}{2}$, which means the general solution for holes is $e^{k\pi} * e^{\frac{\pi}{2}}$

22. **D** : Through partial fraction decomposition, $\frac{3x^2+20x+27}{x^3+10x^2+27x+18} = \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{x+6}$. This means $f(\log_2 x) = \frac{1}{\log_2 x+1} + \frac{1}{\log_2 x+3} + \frac{1}{\log_2 x+6} = \frac{1}{\log_2 2x} + \frac{1}{\log_2 8x} + \frac{1}{\log_2 64x} = \log_{2x} 2 + \log_{8x} 2 + \log_{64x} 2$. Therefore $a + b + c = 74$

23. **C** : $(e^{2\pi i} + e^{\frac{\pi i}{2}})^{2025} = (1 + i)^{2025} = (\sqrt{2})^{2025}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}})^{2025} = (\sqrt{2})^{2025}(e^{\frac{2025\pi}{4}}) = (\sqrt{2})^{2025}(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}) = 2^{1012} + 2^{1012}i$

24. **A** : $f(x) = \log_{10}(\sum_{n=0}^{x}(2^n) + \sum_{n=1}^{\infty}(\frac{1}{2^n}))$ simplifies to $\log_{10}(2^{x+1}) = (x + 1)\log_{10} 2$. Although this function is linear, due to the summation, the domain only includes non-negative integers, so the function is one-to-one but not onto.

25. **D** : All roots such that $x^{12} = 1$ or $x^{16} = 1$ are also roots of $x^{144} = 1$. The least common multiple of 12 and 16 is 48, meaning there are 4 roots such that both $x^{12} = 1$ and $x^{16} = 1$. This means the probability is $\frac{12+16-4-4}{144} = \frac{5}{36}$

26. **D** : To solve this problem, n must be found such that $log_n x = x$ and $\frac{1}{x\ln n} = 1$. This means that $n = \sqrt[x]{x}$ and $\frac{1}{x} = \ln n$, or $e^{\frac{1}{x}} = n$, meaning $x = e$ and $n = \sqrt[e]{e}$

27. **A** : $\ln(\cos 6x) - \ln(\sqrt{2}\cos x - 1) - \ln(\sqrt{2}\cos x + 1) = \ln(4\cos^3 2x - 3\cos 2x) - \ln(2\cos^2 x - 1) = \ln(4\cos^3 2x - 3\cos 2x) - \ln(\cos 2x) = \ln(4\cos^2(2x) - 3) = \ln(2\cos(4x) - 1) = \ln(\sqrt{3} - 1)$. This means that $2\cos(4x) - 1 = \sqrt{3} - 1$ and $\cos(4x) = \frac{\sqrt{3}}{2}$ However, the initial expression has $\cos 6x$, which is negative for $(\frac{\pi}{12}, \frac{\pi}{4})$, and $\sqrt{2}\cos x - 1$, which is negative for $(\frac{\pi}{4}, \frac{7\pi}{4})$. Since the answer can only be in the interval $(0, \frac{\pi}{12})$, the only answer is $\frac{\pi}{24}$.

28. **C** : $\sum_{n=1}^{2025}(\frac{e^{\frac{n\pi i}{2}}}{n} - \frac{ie^{\frac{n\pi i}{2}}}{n+1}) = \sum_{n=1}^{2025}(\frac{e^{\frac{n\pi i}{2}}}{n} - \frac{e^{\frac{n\pi i+\pi i}{2}}}{n+1})$, meaning this is a telescoping series, and equals $e^{\frac{\pi i}{2}} - \frac{e^{2026\pi i}}{2026} = i + \frac{1}{2026}$

29. $\boxed{\text{C}}$: Given that the function is even, it can be proven that the function is always increasing for $(0, \infty)$ and always decreasing for $(-\infty, 0)$, by observing a similar function, $\frac{\sqrt{e^{2x}-2e^x+1}}{e^x+1} = \frac{e^x-1}{e^x+1} = \frac{1}{e^{-x}+1} - \frac{1}{e^x+1}$. The terms $\frac{1}{e^{-x}+1}$ and $-\frac{1}{e^x+1}$ are always increasing, meaning $\frac{\sqrt{e^{2x}-2e^x+1}}{e^x+1}$ is always increasing for $(0, \infty)$, and thus $\frac{\sqrt{e^{2x}-e^x+1}}{e^x+1}$ has a minimum at $x = 0$ and converges to a maximum value as it approaches infinity. These values are $\frac{1}{2}$ and 1.

30. $\boxed{\text{B}}$: $\frac{1}{n} = \sqrt{\frac{f(n-1)}{f(n)}} * \sqrt[4]{\frac{f(n-1)}{f(n)}} * \sqrt[8]{\frac{f(n-1)}{f(n)}} * \sqrt[16]{\frac{f(n-1)}{f(n)}} * ... = \frac{f(n-1)}{f(n)}$, and $f(n) = nf(n-1)$. Since $f(0) = 1$, this is a factorial. $\sum_{n=0}^{\infty}(\frac{1}{n!}) = e$, but since the question asks for all natural numbers, 0 is not included. This means $\sum_{n=1}^{\infty}(\frac{1}{n!}) = e - 1$