

Assume principle argument of $(-\pi, \pi]$ for all complex numbers unless stated otherwise. Good luck, have fun, and as always: "NOTA" stands for "None of These Answers is correct."

- The expression $\frac{xy^3z^2}{x^{\frac{1}{4}}\sqrt{z^{-\frac{1}{3}}y^{-\frac{1}{13}}}}$ may be simplified into the form $x^a y^b z^c$. Find abc

(A) 2 (B) 5 (C) 8 (D) 11 (E) NOTA
- Find the sum of all possible values of x for the equation $\log_{10} x - 6 \log_x 10 = -1$

(A) 1000.01 (B) 1000010 (C) 110 (D) 100.001 (E) NOTA
- Given that $f(x) = \sqrt{1 - (\log_x 10)^2}$ and $g(x) = f(-\frac{1}{x})$, what is the domain of $g(x)$?

(A) $[-10, 0) \cup (0, -\frac{1}{10}]$ (C) $[-1, 0) \cup (0, 10]$ (E) NOTA
(B) $[-10, 0)$ (D) $(-\infty, -10) \cup [-\frac{1}{10}, 0)$
- Given the equation $e^{a^b} = (e^a)^b$, solve for a in terms of b .

(A) $(b-1)^{\log_b e}$ (C) $\log_b(b-1)$ (E) NOTA
(B) $b^{-1}\sqrt{b}$ (D) b^{b-1}
- Find the sum of the values of x where the real parts of the equations e^{ix} and $\frac{e^{2ix}}{\sqrt{e^{\pi i}}}$ for $0 < x < 2\pi$ are equal.

(A) 3π (B) $\frac{5\pi}{2}$ (C) 2π (D) $\frac{3\pi}{2}$ (E) NOTA
- Is the function $f(x) = \sin(x)e^{-\ln(x^2+1)}$ even or odd?

(A) Even (B) Odd (C) Both (D) Neither (E) NOTA
- Find the sum of all possible values of x for the equation $11^{x+1} + 30 \cdot 11^{1-x} = 121$

(A) $\log_{11} 22$ (B) $\log_{11} 24$ (C) $\log_{11} 30$ (D) $\log_{11} 33$ (E) NOTA
- Hyperbolic functions are the counterpart of trigonometric functions, functioning on a hyperbola rather than a circle, and are defined by appending 'h' to the corresponding trigonometric functions. The formulas for $\sinh$ and $\cosh$ are $\sinh x = \frac{e^x - e^{-x}}{2}$ and $\cosh x = \frac{e^x + e^{-x}}{2}$. If $\tanh x = \frac{\sinh x}{\cosh x}$, what is $\frac{\tanh(-x)}{\sinh x + \cosh x}$

(A) $\frac{1-e^{2x}}{e^x+e^{3x}}$ (B) $\frac{e^{3x}-e^x}{1+e^{2x}}$ (C) $\frac{1+e^{2x}}{e^{3x}-e^x}$ (D) $\frac{-2e^x}{e^{-2x}+1}$ (E) NOTA

9. What is the maximum value of $\frac{3^{-x^2} \cdot 9^{3x}}{27}$?

- (A) 3^3 (B) 3^6 (C) 3^9 (D) 3^{12} (E) NOTA

10. Find the domain of $\frac{\ln(x^2+2x-3)+\log_{x+3}(e)}{\ln(x-3)}$

- (A) $(3,4) \cup (4,\infty)$
 (B) $(1,4) \cup (4,\infty)$
 (C) $(-3,1) \cup (4,\infty)$
 (D) $(-\infty,-3) \cup (4,\infty)$
 (E) NOTA

11. $f(x,y) = y^{\log_3 25} + x^{\log_4 36}$

Find the number of points at which $f(x,y)$, x , and y are rational for $1 \leq x, y \leq 1000$

- (A) 96 (B) 70 (C) 48 (D) 36 (E) NOTA

12. Which of the following is NOT an approximation for $e^x + e^{-x}$ near 0?

- (A) $x^2 + 2$ (B) 2 (C) $\frac{x^4}{12} + x^2 + 2$ (D) $\frac{x^3}{3} + x^2 + x + 2$ (E) NOTA

13. Given that $a = \frac{\log_{10} x}{\ln(10x)}$ and $b = \frac{1}{\ln(10) \cdot \log_x \frac{x^2}{10}}$, what is $\frac{ab}{a+b}$

- (A) $\ln(10e^3)$ (B) $(\ln 10)^3$ (C) $\ln(\sqrt[3]{10})$ (D) $\frac{1}{\ln 1000}$ (E) NOTA

14. Find all the holes in the following function $\ln(x^2 - 1) - \ln\left(\frac{x^2+4x-5}{x+1}\right) + \ln\left(\frac{x^2+\frac{7}{2}x-\frac{15}{2}}{x-\frac{3}{2}}\right)$

- (A) $\{-5\}$ (B) $\{\frac{3}{2}\}$ (C) $\{-1, 1, \frac{3}{2}\}$ (D) $\{-5, -1, \frac{3}{2}\}$ (E) NOTA

15. If $a = \ln 2$ and $4^x = 2e$, define x in terms of a

- (A) $\frac{2}{a}$ (C) $\frac{1}{a+1} - 2$ (E) NOTA
 (B) $\frac{1}{2a} + \frac{1}{2}$ (D) $\frac{2a}{a+1}$

16. Evaluate $\lim_{x \rightarrow \infty} \left(1 + \frac{i}{x}\right)^{\pi x}$

- (A) 1 (B) i (C) -1 (D) $-i$ (E) NOTA

17. Find the domain of $\frac{1}{\sqrt{\ln(\sqrt{x^2+x})}}$

- (A) $(-\infty, -1) \cup (0, \infty)$
 (B) $(-\infty, \frac{-1-\sqrt{5}}{2}) \cup (\frac{-1+\sqrt{5}}{2}, \infty)$
 (C) $(-\infty, \frac{-1-\sqrt{1+4e}}{2}) \cup (\frac{-1+\sqrt{1+4e}}{2}, \infty)$
 (D) $(-\infty, \frac{-1-\sqrt{1+4\sqrt{e}}}{2}) \cup (\frac{-1+\sqrt{1+4\sqrt{e}}}{2}, \infty)$
 (E) NOTA

Please Use the Following Information to Answer Questions 18 to 19: The decibel scale is a relative unit of measurement given by the formula $10 \log_{10}(\frac{P_1}{P_2})$ for two sounds of power P_1 and P_2 equidistant from the receiver.

18. A conductor is calculating the optimal amount of violin players in an orchestra. The conductor wants the volume of the violins and flutes in the orchestra to be equal at a particular moment where all instruments are playing. For reference, 8 violins playing simultaneously are 15 dB louder than a single flute. How many violins does the conductor need if there are 20 flutes in the orchestra? (Hint: $10^{\frac{1}{2}} \approx 3.2$)

- (A) 5 (B) 15 (C) 20 (D) 35 (E) NOTA

19. The power of a signal is given by the formula $\frac{V^2}{R}$, where V is the voltage, and R is resistance. If the amplitude of the voltage of a signal over time is given by $13.75 + 8.75 \sin t$, what is the approximate difference, in decibels, between its loudest and quietest? (Hint: $\log_{10} 2025 \approx 3.306$, and resistance is constant)

- (A) 13.06 (B) 15.53 (C) 16.94 (D) 30.6 (E) NOTA

20. Given m , for what values of n does $\log_{(x-m)}(x-n)$ have a vertical asymptote?

- (A) $(-\infty, m) \cup (m, \infty)$
 (B) $(-\infty, m+1) \cup (m+1, \infty)$
 (C) $(m-1, m+1)$
 (D) $(-\infty, m) \cup (m+1, \infty)$
 (E) NOTA

21. What is the general solution for hole locations in the function $\cos^2(\frac{1}{\cos(\ln x)}) + \sin^2(\frac{1}{\cos(\ln x)})$? In each answer choice, k is an integer.

- (A) $x = e^{\frac{k\pi}{2}} \cdot e^{\frac{1}{2}}$ (C) $x = e^{\frac{k\pi}{4}}$ (E) NOTA
 (B) $x = e^{2k\pi} \cdot e^{\frac{\pi}{2}}$ (D) $x = e^{\frac{k\pi}{2}} \cdot e^{\frac{\pi}{2}}$

22. If $f(x) = \frac{3x^2+20x+27}{x^3+10x^2+27x+18}$ and $f(\log_2 x) = \log_{ax} 2 + \log_{bx} 2 + \log_{cx} 2$, what is $a + b + c$?
- (A) 44 (B) 50 (C) 70 (D) 74 (E) NOTA
23. Evaluate $(e^{2\pi i} + e^{\frac{\pi i}{2}})^{2025}$
- (A) 2^{1012} (C) $2^{1012} + 2^{1012}i$ (E) NOTA
(B) $\sqrt{2}^{2025} + \sqrt{2}^{2025}i$ (D) $2^{2025} + 2^{2025}i$
24. Evaluate $\lim_{x \rightarrow \infty} \frac{\log_{10}(9 \sum_{n=0}^x (10^n) + 9 \sum_{n=1}^{\infty} (\frac{1}{10^n}))}{x+9}$
- (A) 1 (B) 9 (C) 10 (D) 19 (E) NOTA
25. A random root is selected such that $x^{144} = 1$. What is the probability that either $x^{12} = 1$ or $x^{16} = 1$, but not both.
- (A) $\frac{1}{8}$ (B) $\frac{7}{48}$ (C) $\frac{11}{72}$ (D) $\frac{5}{36}$ (E) NOTA
26. Find n such that $y = \log_n x$ is tangent to $y = x$ (Hint: The slope of a line tangent to the curve at position x is $\frac{1}{x \ln n}$)
- (A) e^e (B) e^2 (C) e^{-e} (D) $\sqrt[e]{e}$ (E) NOTA
27. For x in range $(0, \frac{3\pi}{2})$ what is the sum of the possible values of x for $\ln(\sqrt{3} - 1) = \ln(\cos 6x) - \ln(\sqrt{2} \cos x - 1) - \ln(\sqrt{2} \cos x + 1)$
- (A) $\frac{\pi}{24}$ (B) $\frac{\pi}{3}$ (C) 2π (D) $\frac{25\pi}{3}$ (E) NOTA
28. Evaluate $\sum_{n=1}^{2025} \left(\frac{e^{\frac{n\pi i}{2}}}{n} - \frac{ie^{\frac{n\pi i}{2}}}{n+1} \right)$
- (A) $i + \frac{1}{2025}$ (C) $i + \frac{1}{2026}$ (E) NOTA
(B) $i - \frac{1}{2025}$ (D) $i - \frac{1}{2026}$
29. What is the sum of the bounds of the range of $\frac{\sqrt{e^{2x}-e^x+1}}{e^x+1}$? (Hint: This function is even.)
- (A) 0 (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) NOTA
30. Given that $\frac{1}{n} = \sqrt{\frac{f(n-1)}{f(n)}} \cdot \sqrt[4]{\frac{f(n-1)}{f(n)}} \cdot \sqrt[8]{\frac{f(n-1)}{f(n)}} \cdot \sqrt[16]{\frac{f(n-1)}{f(n)}} \cdot \dots$, find the sum of the reciprocals of $f(n)$ for all natural numbers if $f(0) = 1$.
- (A) $\sqrt{e}$ (B) $e - 1$ (C) $2e$ (D) $\frac{1}{e}$ (E) NOTA