History of Mathematics
"Proofs and Theorems"
Open, Round 2
Test #402

1. Write your 6-digit ID# in the I.D. NUMBER grid, left-justified, and bubble. Check that each column has only one number darkened.
2. In the EXAM NO. grid, write the 3-digit Test # on this test cover and bubble.
3. In the Name blank, print your name; in the Subject blank, print the name of the test; in the Date blank, print your school name (no abbreviations).
4. Scoring for this test is 5 times the number correct + the number omitted.
5. You may not sit adjacent to anyone from your school.
6. TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.
7. No calculators may be used on this test.
8. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
9. If a student believes a test item is defective, select "E) NOTA" and file a Dispute Form explaining why.
10. If a problem has multiple correct answers, any of those answers will be counted as correct. Do not select "E) NOTA" in that instance.
11. Unless a question asks for an approximation or a rounded answer, give the exact answer.

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Statement "A" is of the form "If X, then Y." If Statement A has been proven to be true, which of the following is also necessarily true?

   (A) A's Inverse (B) A's Converse

   (C) The Inverse of A's Converse (D) The Converse of A's Contrapositive

   (E) NOTA

2. All of the following produced examples of Pythagorean Triples before 1 C.E. EXCEPT the

   (A) Babylonians (B) Indians (C) Chinese (D) Europeans (E) NOTA

3. Bertrand's Postulate was first proven to be true by

   (A) Pafnuty Chebyshev (B) Paul Erdős

   (C) Srinivasa Ramanujan (D) Bertrand Russell (E) NOTA

4. The Seven Bridges of Königsberg Problem (which was proven false by Euler) highly influenced the creation of a new branch of mathematics. This branch was

   (A) Analytic Geometry (B) Number Theory

   (C) Graph Theory (D) Enumerative Combinatorics (E) NOTA

5. Proofs that have used recursion must also have used

   (A) Contradiction (B) Induction

   (C) Transposition (D) Construction (E) NOTA

6. Euler used the following to prove that $e^{ix} = \cos x + i \sin x$, where $i = \sqrt{-1}$.

   (A) Trigonometric Analysis (B) e^{ix} expressed in polar coordinates.

   (C) Limits (D) Power Series (E) NOTA

7. The proof of the Mean Value Theorem is based on

   (A) The Five Lemma (B) Rolle's Theorem

   (C) Liouville's Theorem (D) Burnside's Lemma (E) NOTA

8. Georg Cantor's mathematical proofs were primarily in the field of

   (A) Linear Algebra (B) Riemann Geometry

   (C) Vector Analysis (D) Set Theory (E) NOTA

9. All of the following Millenium Prize Problems remain unsolved as of the administration of this test, EXCEPT for

   (A) P vs. NP (B) Hodge Conjecture

   (C) The Riemann Hypothesis (D) The Yang-Mills Existence and Mass Gap

   (E) NOTA

10. The first widely accepted proof of the Four Color Theorem was given in

    (A) 1976 C.E. (B) 1215 C.E. (C) 2005 C.E. (D) 1879 C.E. (E) NOTA

11. The first correct proof of Fermat's Last Theorem was given by

    (A) Leonhard Euler (B) John Milnor

    (C) Ian Stewart (D) Andrew Wiles (E) NOTA

12. The Brahmagupta Theorem is in the field of

    (A) Algebra (B) Calculus (C) Geometry (D) Number Theory

    (E) NOTA

13. De Moivre's Formula is the direct result of

    (A) The Hodge Conjecture (B) Euler's Formula

    (C) Rolle's Theorem (D) Cayley's Theorem (E) NOTA

14. The number π was proven irrational in which century?

(A) 4th C.E. (B) 7th C.E. (C) 4th C.E. (D) 14th C.E. (E) NOTA

15. The fact that the sum of the reciprocals of the positive prime numbers is a divergent series was first proven by

(A) Euclid (B) Descartes (C) Cantor (D) Fibonacci (E) NOTA

16. The famous Incompleteness Theorem of axiomatic systems able to perform arithmetic was first proven by

(A) Leonhard Euler (B) Bertrand Russell

(C) Kurt Gödel (D) Pafnuty Chebyshev (E) NOTA

17. Many of Euler and Lagrange's proofs are considered illegitimate by modern standards because they are not mathematically rigorus. This is because in their proofs, they used

(A) Improper proof checking. (B) Unproven foundational theorems.

(C) The generality of algebra. (D) The Fundamental Theorem of Algebra

(E) NOTA

18. Introduced in 1760, Euler developed a function, often denoted as $\varphi(n)$, which is often called his

(A) Totient Function (B) Graph Theory Function

(C) Durante Function (D) Kummer Function (E) NOTA

19. The fact that the square of the period of a planet's orbit is proportional to the cube of its semi-major axis, or $p^2 \propto a^3$, was first conjectured by

(A) Leonhard Euler (B) Albert Einstein

(C) Tycho Brahe (D) Johannes Kepler (E) NOTA

20. The first person to prove that the Diophantine approximation of π overestimates π was

(A) Fibonacci (B) Archimedes (C) Euclid (D) Diophantine (E) NOTA

21. The first general and complete proof to the quadratic formula comes from

(A) India (B) China

(C) The Middle East (D) Europe (E) NOTA

22. Lagrange's Four-Square Theorem is also known as

(A) Fermat's Theorem on the Sums of Two Squares

(B) Ito's Lemma

(C) Rice's Theorem

(D) Bachet's Conjecture

(E) NOTA

23. In 1736, Euler published a proof of Fermat's Little Theorem, but another mathematician had written almost the same proof in an unpublished manuscript over fifty years earlier. This other mathematician was

(A) Gottfried Leibniz (B) Isaac Newton

(C) Robert Boyle (D) Rene Descartes (E) NOTA

24. This text contained the first proof of the Chinese Remainder Theorem

(A) The Mathematical Classic of Sun Zi

(B) Jigu Suanjing

(C) Mathematical Treatise in Nine Sections

(D) Ceyuan Haijing

(E) NOTA

25. Leonhard Euler, perhaps best known for Euler's Identity, Euler's Formula, and Euler's Number, wrote many proofs in the field of mathematics. In his works he introduced all of the following common notations EXCEPT

(A) e (B) π (C) Σ (D) i (E) NOTA

26. Georg Cantor stated that any definable collection is a set. This was proven to be false by "The set containing all sets that are not members of themselves." The man who posed this paradox was

(A) John Forbes Nash (B) Robert Boyle

(C) Leonhard Euler (D) Bertrand Russell (E) NOTA

27. "Every integer greater than 2 can be expressed as the sum of two primes." This statement has been shown true for numbers at least as high as 4×10^{18} using brute force methods, but no mathematical proof exists as of this writing. This conjecture is called

(A) Cayley's Conjecture (B) The Riemann Hypothesis

(C) Goldbach's Conjecture (D) Waring's Problem (E) NOTA

28. Bernstein's Problem about minimal surfaces was first solved by

(A) Kurt Gödel (B) Donald Knuth

(C) John von Neumann (D) Donald Samuelson (E) NOTA

29. Of the 23 published Hilbert Problems, how many have been definitively solved?

(A) Between 2 and 7, inclusive. (B) Between 8 and 9, inclusive.

(C) Between 10 and 12, inclusive. (D) Between 16 and 19, inclusive. (E) NOTA

30. Paul Erdős was possibly one of the most prolific mathematical writer to ever live, publishing over 1500 papers. He believed that all of the most beautiful and elegant proofs could be found in which of the following?

(A) SamLand (B) The Book (C) The Principia

(D) The Epsilons (E) NOTA