Probability/Permutations/Combinations - Theta
Mu Alpha Theta National Convention 2003

For all questions, answer E "NOTA" means none of the above answers is correct.

1. There are 15 people in Mr. Smith's Algebra II math team. His principal asks for 2 representatives from the team to serve on a committee. How many different delegations can Mr. Smith send?
A. 15! B. ${}_{15}P_2$ C. ${}_{15}C_2$ D. (15)(2) E. NOTA

2. A small engineering firm employs 25 engineers and 10 sales representatives. A committee of 3 engineers and 2 sales representatives is to be formed to discuss a new product. How many different committees are possible?
A. 13890 B. 103500 C. 324632 D. 1242000 E. NOTA

3. In a certain state, license plates consist of a prefix indicating the issuing county, 2 digits, and 3 letters. Assuming all license plates are of this form, what is the maximum number of license plates that can be issued by one county in this state?
A. 1757600 B. 1423656 C. 1404000 D. 1265625 E. NOTA

4. Find ${}_nP_3$ if ${}_nC_3 = 12n$.
A. 720 B. 210 C. 120 D. 35 E. NOTA

5. The odds against event A occurring are 9:7. Find the probability that A occurs.
A. $\frac{7}{9}$ B. $\frac{9}{16}$ C. $\frac{7}{16}$ D. $\frac{2}{7}$ E. NOTA

6. In how many distinct ways can the letters in MUALPHATHETA be arranged?
A. 479001600 B. 79833600 C. 39916800 D. 19958400 E. NOTA

7. Find the number of diagonals in a convex decagon.
A. 10 B. 18 C. 28 D. 35 E. NOTA

8. Three fair dice are rolled. Find the probability that the faces showing total at most 16.
A. $\frac{1}{108}$ B. $\frac{1}{54}$ C. $\frac{103}{108}$ D. $\frac{53}{54}$ E. NOTA

9. How many different odd integers can be formed using the digits 2, 3, 5, and 7 if no digit can repeat?
A. 18 B. 27 C. 36 D. 45 E. NOTA

10. The set S is {#, !, @, *, $, %}. How many different proper subsets of S are possible?
A. 6 B. 63 C. 64 D. 127 E. NOTA

11. Find the constant term of the expansion of $\left(x - \frac{2}{x^2}\right)^9$.
A. 672 B. 84 C. 1 D. –84 E. NOTA

12. If a positive integral factor of 2010 is chosen at random, find the probability that the factor is even.
A. $\frac{1}{4}$ B. $\frac{5}{8}$ C. $\frac{3}{4}$ D. $\frac{7}{16}$ E. NOTA

13. In a group of 10 chihuahuas, 5 are female and 3 are black. If one-fifth of the dogs are black males, what is the probability that a dog chosen at random will be a female that is not black?
A. $\frac{1}{10}$ B. $\frac{1}{5}$ C. $\frac{3}{10}$ D. $\frac{2}{5}$ E. NOTA

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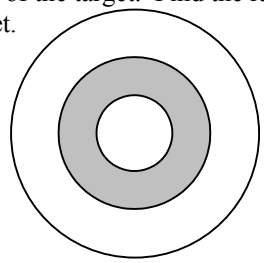
14. If $\binom{a}{5} = \binom{a}{7}$ and $b = {}_aP_2$, find $a + b$.

A. 4 B. 78 C. 144 D. 1325 E. NOTA

15. A target consists of three concentric circles, as pictured below. If a dart lands on the target, it is equally likely to land in any one of the three sections of the target. Find the ratio of the length of the shortest radius of the target to that of the middle radius of the target.



A. 1:4 B. 1:2 C. 2:3 D. $1:\sqrt{2}$ E. NOTA

16. If x and y are two positive numbers greater than 0 but less than 4, what is the probability that the sum of x and y is less than 2?

A. $\frac{1}{8}$ B. $\frac{1}{6}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. NOTA

17. A standard deck of 52 cards, in which half are red and half are black, contains 4 suits clubs (black), spades (black), hearts (red) and diamonds (red). Each suits contains 13 cards, numbered from 1 (the ace) to 10, plus a king, a queen, and a jack.
One card is drawn from a well-shuffled standard deck of 52 cards. What is the probability that the card is a three or a king?

A. $\frac{1}{52}$ B. $\frac{1}{26}$ C. $\frac{1}{13}$ D. $\frac{2}{13}$ E. NOTA

18. The names of 18 regular polygons are written on index cards, one polygon per card. The names are equilateral triangle, square, regular pentagon, regular hexagon, and so on until the eighteenth card, which says regular icosagon (20 sides). One card is selected at random from the index cards. What is the probability that the card will **not** have the name of a regular polygon with integral interior degree measures ?

A. $\frac{7}{20}$ B. $\frac{1}{2}$ C. $\frac{11}{20}$ D. 1 E. NOTA

19. For one variation of the card game ROOK, you use a 45 card deck. There is 1 Rook card and four suits of 11 cards each. The suits are red, yellow, green, and black. Players are dealt 10 cards each, and the remaining 5 cards are placed in a hand that is not used until later in the game. If the Rook is considered to be of any suit, in how many ways can a player be dealt a hand with all 10 cards in the same suit?

A. 44 B. 66 C. 264 D. 528 E. NOTA

20. In how many different ways can first, second, and third bases be occupied in a game of baseball. For example, one configuration is runners at first and second and third base is empty.

A. 3 B. 4 C. 6 D. 8 E. NOTA

21. A weather forecaster claims a 40% chance that a hurricane will hit the coast next week. If it does, there is a 10% probability that the damage will be severe. What is the probability that the storm hits and the damage is severe?

A. 4% B. 25% C. 30% D. 50% E. NOTA

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22. The base 10 numbers 1 through 10,000 inclusive are written on index cards, one number per card. One card is chosen at random. Find the probability that the card chosen shows a number with at least three digits when converted to base 6.

A. $\frac{99}{100}$ B. $\frac{1993}{2000}$ C. $\frac{2491}{2500}$ D. $\frac{1957}{2000}$ E. NOTA

23. Six of the letters in the word BOGGLES are chosen and arranged. How many distinct arrangements are possible?

A. 5040 B. 4680 C. 3240 D. 2520 E. NOTA

24. L is the set of all lattice points (points with integral coordinates) on the graph of $x^2 + y^2 = 625$. Four points are selected at random from L to be the vertices of a quadrilateral. Find the probability that a parallelogram is formed by the set chosen.

A. $\frac{4}{495}$ B. $\frac{2}{495}$ C. $\frac{4}{4845}$ D. $\frac{2}{4845}$ E. NOTA

25. Find the number of different arrangements of the letters in the word PRIVACY if all the vowels must be placed together. Consider Y as a consonant.

A. 5040 B. 1440 C. 1008 D. 720 E. NOTA

26. Student ID numbers at Mediocre U are 9 digits long where each digit ranges from 0 to 9 inclusive, and each student has a unique ID #. From all possible Mediocre ID numbers, one is selected at random. What is the probability that the number chosen has distinct digits arranged in increasing, consecutive order?

A. $\frac{1}{10^9}$ B. $\frac{1}{10!}$ C. $\frac{2}{10!}$ D. $\frac{2}{{}_{10}C_9}$ E. NOTA

27. The 500 lockers at Gen High are assigned one student per locker. All of the locker doors are shut at the beginning of the day. The student assigned to locker 1 opens all 500 lockers. Then the student assigned to locker 2 closes the doors of all lockers numbered with a multiple of 2. The student assigned to locker 3 then reverses all lockers numbered with a multiple of 3 (if the locker's open, student 3 closes it and vice versa), the student assigned to locker 4 then reverses all lockers numbered with a multiple of 4, and so on until the student assigned to locker 500 reverses that locker. At the end of the day, how many lockers will be left open?

A. 22 B. 94 C. 95 D. 96 E. NOTA

28. A high school has 1200 students, of whom 640 are young women. The junior class has 360 students, of whom 200 are young women. What is the probability that a student picked at random is either a junior or a young woman?

A. $\frac{3}{10}$ B. $\frac{8}{15}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

29. A box contains 4 red chips and 2 blue chips. Two chips are drawn simultaneously.
Find the probability that the chips have different colors.

A. $\frac{4}{15}$ B. $\frac{6}{15}$ C. $\frac{8}{15}$ D. 1 E. NOTA

30. On a celebrity match of the Weakest Link, Big Bird and Barney make it to the final round. The final round consists of five questions for each player. Play is stopped if it becomes evident that one of the contestants cannot possibly win. If Big Bird has answered his questions correctly 75% of the time and Barney has answered his questions correctly 60% of the time, what is the probability that the final round will end after each player has answered exactly four questions?

A. $\frac{3753}{160000}$ B. $\frac{2943}{40000}$ C. $\frac{3753}{40000}$ D. $\frac{3753}{32000}$ E. NOTA