SEQUENCES AND SERIES - CALCULUS
Mu Alpha Theta National Convention 2003

For all questions, E. "NOTA" means none of the above answers are correct.

1. In a training regimen, an athlete jogs 1 mile on day 1. On each successive day, the athlete jogs 10% further than the prior day. What is the first day, to the nearest day, that the athlete will jog at least 5 miles?

A. 4 B. 5
C. 17 D. 18 E. NOTA

2. Determine the geometric mean of 175 and 567.

A. 315 B. 371
C. 49,612.5 D. 99,225 E. NOTA

3. The sequence 3, 5, 7, 9 is repeated 10 more times (a total of 44 numbers). What is the sum of all the terms of this sequence?

A. 240 B. 264
C. 1,056 D. 1,892 E. NOTA

4. In a twenty-term arithmetic sequence, the final term is 79 and the sum of the terms is 820. What is the sequence's common difference?

A. 2 B. 3
C. 4 D. 5 E. NOTA

5. If $\cos\theta + \cos\theta \sin\theta + \cos\theta \sin^2\theta + ... = \sqrt{3}$ then θ could be which of the following?

A. $\frac{\pi}{6}$ B. $\frac{\pi}{4}$

C. $\frac{\pi}{3}$ D. $\frac{5\pi}{6}$ E. NOTA

6. Determine the following sum:
$(-17)^2 + (-16)^2 + (-15)^2 + ... + 15^2 + 16^2 + 17^2 + 18^2$.

A. 324 B. 3,570
C. 3,894 D. 4,218 E. NOTA

7. Determine the common ratio of an infinite geometric series whose first term is $(4+i)$ and whose sum is $(3+5i)$. Note: $i = \sqrt{-1}$.

A. $1 - i$ B. $1 + i$

C. $\frac{-1}{2} + \frac{-1}{2}i$ D. $\frac{1}{2} + \frac{1}{2}i$ E. NOTA

8. On Wednesday, January 1, 2003, Helene watches a movie. She watched a movie on January 4, 7, 10, etc., and continues this pattern for three years. A Tuesday in 2003 is randomly selected from a calendar. What is the probability that Helene saw a film on the chosen day? (Note: 2003 is not a leap year).

A. $\frac{1}{7}$ B. $\frac{17}{52}$

C. $\frac{9}{26}$ D. $\frac{27}{52}$ E. NOTA

9. $\sum_{n=3}^{12}\left(4 \bullet 2^n + 6n\right) = ?$

A. 33,141 B. 33,186.
C. 33,228 D. ∞ E. NOTA

10. $\sqrt{13+\sqrt{5+\sqrt{13+\sqrt{5+\ldots}}}} = ?$

A. No Solution B. 4
C. 8 D. 12 E. NOTA

11. Determine the value of z: $\sum_{i=1}^{z}\begin{vmatrix} 3i & 5 \\ 1 & 2 \end{vmatrix} = 40.$

A. 4 B. 5
C. 8 D. 10 E. NOTA

12. Which of the following is the coefficient of the 5^{th} term in the expansion of $(1+x)^{\frac{3}{2}}$?

A. $\frac{3}{240}$ B. $\frac{3}{128}$

C. $\frac{3}{64}$ D. $\frac{3}{48}$ E. NOTA

13. Determine the infinite sum:
$\frac{17}{3}+\frac{13}{9}+\frac{9}{27}+\ldots+\frac{21-4n}{3^n}+\ldots.$

A. 2 B. 7.5
C. 15 D. Diverges E. NOTA

14. In a sequence, $a_n = 10^{(a_{n-1}-9)}$ for n=1,2,3…. If $a_1 = 10$ then $a_{10} = ?$

A. 1 B. 10
C. 10^{-10} D. 10^{10} E. NOTA

For numbers 15-18, determine the limit of the sequence as n approaches (positive) infinity.

15. The sequence is: $9, 6\frac{1}{2}, 5\frac{2}{3}, \ldots, 4+\frac{5}{n}, \ldots$.

A. 4 B. 5
C. 9 D. Diverges E. NOTA

16. The sequence is:
$\frac{-\sqrt{10}}{10}, \frac{3\sqrt{145}}{145}, \frac{5\sqrt{730}}{438}, \ldots, \frac{n^3-2}{\sqrt{9n^6+n^2}}, \ldots.$

A. 0 B. $\frac{1}{9}$

C. $\frac{1}{3}$ D. Diverges E. NOTA

17. The sequence is: $\frac{3}{2}, \frac{25}{16}, \frac{343}{216}, \ldots, \left(1+\frac{1}{2n}\right)^n, \ldots$.

A. $\sqrt{e}$ B. $\frac{e}{2}$
C. $2e$ D. Diverges E. NOTA

18. The sequence is: $\frac{\cos(1)-2}{3}, 2\sqrt{2}(\cos(2)-2),$

$3\sqrt{3}(2-\cos(6)), \ldots, \frac{[2-\cos(n!)]n^{\frac{3}{2}}}{2n-5}, \ldots.$

A. 0 B. 0.5
C. 1 D. Diverges E. NOTA

19. The *n*th term of a sequence is the *n*th derivative of $f(x) = (\ln x)^3$ at $x = e^n$ (for n=1,2,3,…). What is the third term of this sequence?

A. $\frac{-6}{e^3}$ B. $\frac{6}{e^6}$

C. $\frac{6}{e^9}$ D. $\frac{18}{e^{15}}$ E. NOTA

20. Determine: $\sum_{n=5}^{\infty} \frac{1}{(n-2)(n+1)}$.

A. $\frac{1}{15}$ B. $\frac{47}{180}$

C. $\frac{47}{60}$ D. Diverges E. NOTA

21. What is the least number of terms in the series $\sum_{n=1}^{\infty} \frac{2(-1)^n}{n^2+4}$ are needed to approximate its sum correctly to four decimal places?

A. 198 B. 199
C. 200 D. 201 E. NOTA

22. Let Q(*b*) return the area (in square units) of a circle inscribed in a square with area *b* square units. To the nearest hundredth, determine the value of $\sum_{p=1}^{\infty} Q\left(\frac{2}{p}\right)$.

A. 0 B. 0.82
C. 1.64 D. Diverges E. NOTA

23. Which is true of the sequence $a_n = \frac{n^n}{n!}$ for n>1?

A. The sequence is everywhere increasing.
B. The sequence is everywhere decreasing.
C. The sequences increases only when $n! < n^n$.
D. No pattern of increase/decrease emerges.
E. NOTA

24. An infinite series has *n*th term equal to $\frac{3n^{\frac{4}{5}}}{3^n}x^n$ for *n*=1,2,3,… . Determine the interval of convergence of this series.

A. (-1,1) B. [-1,1)
C. [-3,3] D. (-3,3) E. NOTA

In numbers 25-28, you are given an infinite series and test of convergence. Choose the answer that reflects the result of the application of that test on the series.

25. $\sum_{n=0}^{\infty} \left(-3n^2 e^{-n^3}\right)$; Integral Test.

A. The series converges because $\lim_{x\to\infty} \left(e^{-x^3} - 1\right)$ does not diverge.
B. The series converges because the terms of the sequence tend to 0.
C. The series diverges because $\lim_{x\to\infty} \left(e^{-x^3} - 1\right)$ is not 0.
D. The series diverges because $\int_0^{\infty} \left(e^{-x^3} - 1\right) dx$ diverges.
E. NOTA

26. $\sum_{x=1}^{\infty} \frac{1}{[\cos(5/x)]^x}$; Root Test.

A. Converges because $\sqrt{[\cos(5/x)]^x}$ is always less than 1.
B. Converges because $\lim_{x\to\infty} \frac{1}{\cos(5/x)}$ converges.
C. Diverges because $\lim_{x\to\infty} \frac{1}{\cos(5/x)}$ is not less than 1.
D. Diverges because $\lim_{x\to 0} \frac{1}{\cos(x+5)}$ is greater than 0.
E. NOTA

27. $\sum_{n=1}^{\infty} n^{e^2} e^{-n^2}$; Ratio Test.

A. Converges because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$ does not diverge.
B. Converges because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$.
C. Diverges because $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| > 0$.
D. Diverges because $\lim_{n\to\infty} \left|\frac{a_n}{a_{n+1}}\right| > 1$.
E. NOTA

28. $\sum_{s=1}^{\infty} \frac{(-1)^s \ln(s^2)}{4^s}$; Alternating Series Test.

A. Converges because the absolute value of the terms of the sequence are decreasing.
B. Converges because the absolute value of each successive term is smaller than the previous term *and* because $\lim_{s\to\infty} \frac{\ln(s^2)}{4^s}$ converges.
C. Diverges because $\lim_{s\to\infty} \frac{\ln(s^2)}{4^s}$ diverges.
D. Diverges because the absolute value of each successive term is not monotonic decreasing.
E. NOTA

29. Determine the MacLaurin series for $f(x) = \cos^2(x) - 1$.

A. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{2n}}{(2n)!} x^{2n}$
B. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1} 2^{2n}}{(2n)!} x^{2n}$
C. $\sum_{n=1}^{\infty} \frac{(-1)^{n} 2^{2n}}{(2n+1)!} x^{2n}$
D. $\sum_{n=1}^{\infty} \frac{(-1)^{n} 2^{2n-1}}{(2n)!} x^{2n}$
E. NOTA

30. Determine the following: $\sum_{i=1}^{20} \left(\int_{i+1}^{i+3} 3x^2 dx \right) = ?$

A. 7,999
B. 9,260
C. 22,780
D. 44,100
E. NOTA