Round 1

Question 1
If today is March 1, 2014, how many days remain until February 1, 2015?

Answer: _____

Question 2
Given that
$\log_2(\log_3(\log_4 x)) = \log_3(\log_2(\log_4 y)) = \log_4(\log_3(\log_2 z)) = 0$, find $x + y + z$.

Answer: _____

Question 3
From a group of boys and girls, 15 girls leave. There are then two boys left for each girl. After this, 45 boys leave. There are then five girls for each boy. How many girls were there in the beginning?

Answer: _____

Question 4
The position of a particle moving along the x-axis is given by $x(t) = 3t^3 - 2t^2 + t - 1$. What is the total distance traveled by the particle from $t = 0$ to $t = 3$?

Answer: _____

Question 5
Of 6,000 apples harvested, every third apple is too small, every fourth apple is too green, and every tenth apple is bruised. The remaining apples are perfect. How many perfect apples are harvested?

Answer: _____

Question 6
The three, two-digit integers 30, 72, and N have the property that the product of any two of them is divisible by the third. What is the value of N?

Answer: _____

Question 7
How many times do the graphs of the parabolas $y = 3x^2 + 5x + 2$ and $y = x^2 - 2x - 8$ intersect?

Answer: _____

Question 8
Evaluate:

$$\lim_{q \to 3} \frac{\left((3+q)^2 - 3(3+q)\right) - (3^2 - 3 \cdot 3)}{q}$$

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum = _____

Round 2

Question 1
Evaluate: $245^2 - 235^2$

Answer: _____

Question 2
For how many positive integer values of n are both $n/5$ and $5n$ equal to positive four-digit integers?

Answer: _____

Question 3
How many of the following series diverge?
i) $\sum_{n=1}^{\infty} \frac{1}{n}$ ii) $\sum_{n=2}^{\infty} \frac{n}{3^n+4}$
iii) $\sum_{n=2}^{\infty} \left(n\left(\frac{1}{3}\right)^n\right)$ iv) $\sum_{n=4}^{\infty} \frac{1}{n \ln n}$

Answer: _____

Question 4
Find the sum of the first fifteen terms of the arithmetic sequence: -4, 1, 6, 11,…

Answer: _____

Question 5
In how many quadrants do points that satisfy $-3y - x < 5$ and $-3x < -4$ lie?

Answer: _____

Question 6
How many positive three-digit numbers are perfect numbers, perfect squares, or perfect cubes?

Answer: _____

Question 7
A dodecahedron has twelve pentagonal faces. A *face diagonal* is a line segment that lies along a face of a polyhedron and connects two nonadjacent vertices. How many face diagonals does a regular dodecahedron have?

Answer: _____

Question 8
How many positive three-digit integers do not have 3 as a digit?

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 3

Question 1
Evaluate: $103^2 - 97^2 + 13^2 + 39^2$

Answer: _____

Question 2
How many solutions x to $2\sin^2 x - \sin x = 1$ exist in the interval $x \in (0,2\pi)$?

Answer: _____

Question 3
Given that t and g are differentiable functions, and that $t(3) = 7, t'(3) = 2, t(6) = 9, t'(6) = 3$, $g(2) = 5$, and $g'(2) = 4$; if $C(x) = t(3x)g(x)$, determine the value of $C'(2)$.

Answer: _____

Question 4
Victor takes a test where he receives five points for a correct answer and loses three points for an incorrect answer. Assuming Victor answers 20 questions on the test, how many questions did he answer correctly if he earned 52 points?

Answer: _____

Question 5
Evaluate $\int_0^{15} \lfloor x \rfloor \, dx$, where $\lfloor x \rfloor$ represents the greatest integer less than or equal to x.

Answer: _____

Question 6
What is the area of the region $|x| + |y| \leq 5$ in the Cartesian Plane?

Answer: _____

Question 7
What is the multiplicative inverse of 14 in modulo 41?

Answer: _____

Question 8
If the base 10 number 365,88A is a multiple of 11, what is the value of the digit A?

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 4

Question 1
How many solutions x to $2\sin^2 x + 3\cos x = 3$ exist in the interval $x \in (0,2\pi)$?

Answer: _____

Question 2
If the graph of $y = f(x) = -x^2 + 2x + q$ has a maximum ordinate value of 6 at $x = 1$, find the value of $f(2)$.

Answer: _____

Question 3
Which Greek mathematician developed a fairly accurate estimate for the circumference of the earth and a "sieve" for determining prime numbers? Answer with the number 1, 2, 3, or 4, corresponding to the correct answer.

1) Archimedes
2) Aristarchus
3) Eratosthenes
4) Euclid

Answer: _____

Question 4
In the expansion of $(2x - y)^{10}$ with like-terms combined, find the coefficient of the x^2y^8 term.

Answer: _____

Question 5
What is the largest integer value of x such that 100! is divisible by 2^x?

Answer: _____

Question 6
Express the following difference as a base 10 number:

$$412_7 - 136_7$$

Answer: _____

Question 7
The number 210 can be written as a sum of consecutive positive integers in several ways. When written as the sum of the greatest possible number of consecutive positive integers, what is the largest of these integers?

Answer: _____

Question 8
Evaluate: $\log_4(16^{500})$

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 5

Question 1
If A is a 5×5 matrix with a determinant of 6, what is the determinant of the matrix $5A$?

Answer: _____

Question 2
Find the greatest common factor of 20! and 200000.

Answer: _____

Question 3
If $a + b = 4$ and $ab = 7$, evaluate: $a^2 + b^2$

Answer: _____

Question 4
If the cost C of producing x widgets is $C = 40\sqrt{x} + \frac{x^2}{400}$, how many widgets need to be produced to minimize the cost per widget. Assume that a fractional number of widgets is allowed.

Answer: _____

Question 5
How many integers x satisfy the following inequality?

$$\left|3 - \frac{2x}{3}\right| < 20$$

Answer: _____

Question 6
How many of the following series converge?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}, \quad \sum_{n=1}^{\infty} \left((n+1)\left(\frac{2}{3}\right)^{n+1}\right), \quad \sum_{n=1}^{\infty} \frac{1}{n^2+n+3},$$

$$\sum_{n=1}^{\infty} \frac{7^{n-1}}{3^{n+1}}, \qquad \sum_{n=1}^{\infty} \sqrt[n]{\frac{3n+1}{4n-3}}$$

Answer: _____

Question 7
Evaluate: $(\log_2 25)(\log_5 8)(\log_3 49)(\log_7 243)$

Answer: _____

Question 8
Evaluate $\sum_{n=1}^{144} \frac{\sqrt{n}}{2}$ and round your answer to the nearest integer.

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 6

Question 1
If the value of the definite integral

$$\int_2^3 \frac{x^4 - 2x^3 + 3x^2 - 2x + 1}{(x^3 - 3x + 1)^2}\,dx$$

can be written as m/n, where m and n are relatively prime positive integers, find $m + n$.

Answer: _____

Question 2
For how many integer values of x does $x^2 + 2x - 19$ have a negative value?

Answer: _____

Question 3
Solve for x: $2^{4x+8} \cdot 4^{2x+3} = 8^{2x+6}$

Answer: _____

Question 4
Evaluate: $\begin{vmatrix} 3 & 1 & 0 & 2 \\ 4 & -1 & -2 & 1 \\ 2 & 2 & -2 & 1 \\ 2 & 3 & 6 & 0 \end{vmatrix}$

Answer: _____

Question 5
If six fair, two-sided coins are flipped, the probability of obtaining more than two heads is m/n, where m and n are relatively prime positive integers. Find $m + n$.

Answer: _____

Question 6
If $\frac{dy}{dx} = 3x^2$ and $y(-1) = 2$, then find the value of

$$\int_0^2 y(x)\,dx.$$

Answer: _____

Question 7
How many ways can six indistinguishable pieces of candy be distributed among three children, provided there is no requirement that each child receive at least one piece of candy?

Answer: _____

Question 8
Find the sum of all positive, two-digit integers that are multiples of 4.

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 7

Question 1
A triangle with vertices at (1,4), (2,7), and $(3,-1)$ is subjected to a linear transformation represented by the matrix $\begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$, resulting in a new triangle. What is the area of this new triangle?

Answer: _____

Question 2
If $\int_0^4 (x^2 - 6x + 9)\, dx$ is approximated by four inscribed rectangles of equal width along the x-axis, what is the value of the approximation?

Answer: _____

Question 3
If $\log_{16} 2 = 1/4$, find the value of $\log_8 4096$.

Answer: _____

Question 4
When listing out the digits of $\lfloor 10^{2013}\pi \rfloor$ from left to right, which of the digits 0 to 9, inclusive, is the last one to make its first appearance?

Answer: _____

Question 5
What number, when added to the numerator and denominator of 5/8, results in a fraction whose value is equal to 0.40?

Answer: _____

Question 6
Find the remainder when 41^{100} is divided by 29.

Answer: _____

Question 7
The Fibonacci Sequence F is defined by $F_1 = F_2 = 1$ and for integers n, $F_{n+2} = F_{n+1} + F_n$. Find the value of $F_{-11} + F_6$.

Answer: _____

Question 8
Find the maximum value of the function $f(x) = 2x^3 - 9x^2 + 12x - 1$ on the interval $x \in [-1,2]$.

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 8

Question 1
In how many ways can 36 be written as a product of the form $abcd$, where a, b, c, and d are positive integers such that $a \leq b \leq c \leq d$?

Answer: _____

Question 2
The first three terms of an arithmetic sequence are $x + 3$, $3x - 1$, and $7x - 3$, in that order. What is the numerical value of the product of the first three terms of the sequence?

Answer: _____

Question 3
Evaluate:

$$\sum_{n=1}^{10} n^3$$

Answer: _____

Question 4
A ball was floating in a lake when the lake froze. The ball was removed without breaking the ice, leaving a hole 24 centimeters across the top and 8 centimeters deep. What was the radius of the ball? Express your answer in centimeters.

Answer: _____

Question 5
The arclength of the graph of $y = x^{3/2}$ on the interval $x \in \left[0, \frac{4}{3}\right]$ is equal to $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Answer: _____

Question 6
In the following system, find $x - y - z$, given that x, y, and z are rational numbers.

$$2^x 3^y 5^z = 7500$$
$$3^x 5^y 2^z = 720$$
$$5^x 2^y 3^z = 4050$$

Answer: _____

Question 7
For how many positive integers n is $n! - 1$ divisible by n?

Answer: _____

Question 8
A cube is made up of 27 fair, six-sided dice. Each die's opposite sides add up to 7. What is the smallest possible sum of all the values visible on the six faces of the large cube?

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 9

Question 1
Two tangent lines can be drawn from the point $(-2,-2)$ to the graph of $y = f(x) = x^2 + 2x + 3$. Find the sum of the x-coordinates of the points where those lines are tangent to the graph of $y = f(x)$.

Answer: _____

Question 2
What is the smallest positive integer that can be expressed as the sum of two positive perfect cubes in exactly two distinct ways?

Answer: _____

Question 3
In triangle ABC, $\tan A = 5/12$ and $\sec B = 5/3$. If $\sin C = m/n$, where m and n are relatively prime positive integers, find $m + n$.

Answer: _____

Question 4
A sequence of functions is defined as $f_1(x) = x - 1$ and $f_n(x) = 2f_{n-1}(x)$ for $n > 1$. Find $f_{10}(2)$.

Answer: _____

Question 5
The repeating decimal $.4\overline{75}$ is equal to m/n, where m and n are relatively prime positive integers. Find $m + n$.

Answer: _____

Question 6
Evaluate: $998^2 - 999^2$

Answer: _____

Question 7
If Kevin invests \$52000 at a 4% annual interest rate compounded continuously, how many years will it take for Kevin's investment to triple? Express your answer to the nearest year.

Answer: _____

Question 8
How many of the following series *diverge*?

$\sum_{n=1}^{\infty} \frac{2}{3^n}$, $\sum_{n=1}^{\infty} \left(\frac{5}{3}\right)^n$, $\sum_{n=1}^{\infty} \frac{2}{3n}$, $\sum_{n=1}^{\infty} \frac{2}{n^3}$, $\sum_{n=1}^{\infty} \frac{2n}{3^n}$

Answer: _____

What is the sum of the answers for Questions 1-8?

Sum= _____

Round 10

Question 1
The ellipse $x^2 + 4y^2 - 4x + 40y = 152$ has major and minor axes lengths of R_1 and R_2, respectively. Find $R_1 + R_2$.

Answer: _____

Question 2
Let $i = \sqrt{-1}$. Given that $(2i + 7)^3 = a + bi$, where a and b are real numbers, find the ratio of a to b. Express your answer to the nearest integer.

Answer: _____

Question 3
For real number x, the expression

$$L = \log_5(\log_4(\log_3(\log_2 x)))$$

is a well-defined real number when $x > c$; otherwise L is not a well-defined real number. Find the value of c.

Answer: _____

Question 4
The area of the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = x^2$ is equal to m/n, where m and n are relatively prime positive integers. Find $n^2 - m^2$.

Answer: _____

Question 5
Let x equal the sum of all two-digit positive prime numbers. Which of the following from 1 to 5 is correct regarding x? Answer with the number that corresponds to the correct answer.

1. x is a prime number.
2. x is a deficient number
3. x is an abundant number.
4. x is a perfect number.
5. None of 1 to 4.

Answer: _____

Question 6
A woman born in the first half of the nineteenth century was x years old in the year x^2. In what year was she born?

Answer: _____

Question 7
A tire with a radius of length two feet travels one mile. How many complete revolutions does the tire make?

Answer: _____

Question 8
The lines $2x - 3y = 4$ and $4x + ky = 13$ are parallel. Find the value of k.

Answer: _____

What is the sum of the answers for Questions 1-8? **Sum=** _____