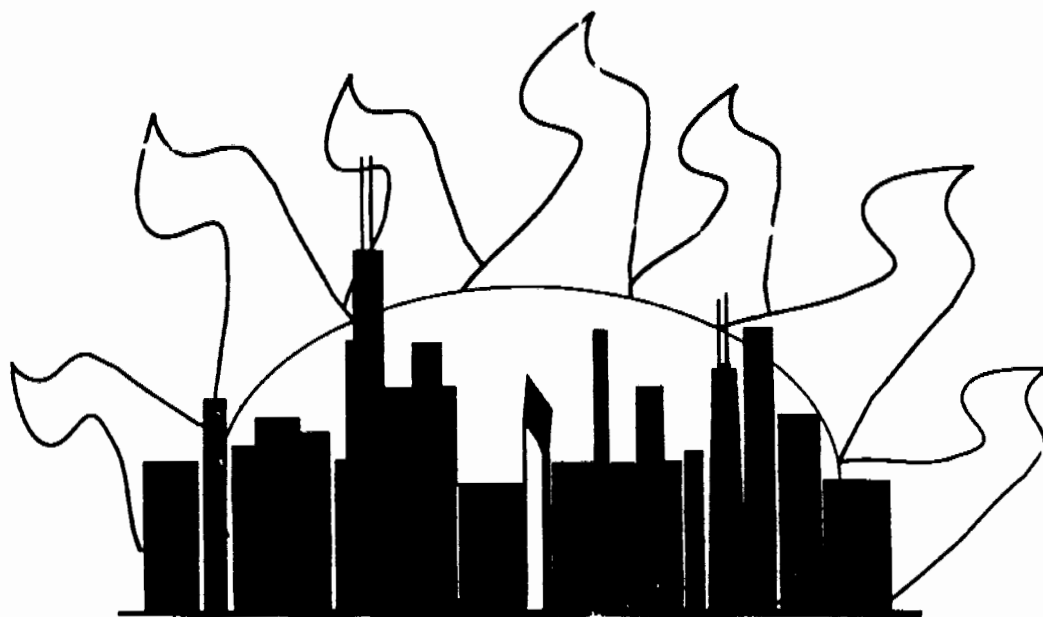# Numeration Systems

Mu Alpha Theta National Convention
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General Instructions:

Unless otherwise stated all answers should be written as decimals.

If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.

Questions

1.   Let $\oplus$ be the binary operation defined by $x \oplus y = \sqrt{xy}$. Find $9 \oplus (3 \oplus 48)$. Give an exact answer.

2.   Find the rational number whose decimal representation is $0.\overline{714285}$. Express your answer as a reduced fraction.

3.   In a numeration system with a positive integral base, the numbers 104 and 241 represent the degree measures of a pair of supplementary angles. Find the base of this numeration system. Write your answer in base ten.

4.   The number $4a35_{\text{seven}} = 159a_{\text{ten}}$. Determine the value of a.

5.   What is the value of 110101.101_{two} in the base ten system? Write your answer as a reduced fraction.

6.   What is the sum of the digits in the product of $9999...99 \times 4444...44$ where there are 98 nines and 98 fours.

7.   If it is now 1 pm, what time will it be in 999,999,999,999,999 hours from now? Write your answer using 24 hour time; i.e. 3 pm = 15.

8.   This computation is done in base B: $(213_B)(32_B) = 12421_B$. Calculate 1021_B divided by 13_B and give your answer in base B.

9.   If it takes one second to write down each digit, how many seconds will it take to write down all the numbers from 1 to 1000?

10. Write 1998 in base eleven. Use x for ten, if necessary.

11. When a number is written in base fifty-one and in base one hundred eighty-seven, the expansion of $1/n$ terminates. If n is an integer greater than 1, find the least possible value of n. Express your answer in base ten.

12. How many positive integers have base four representation with five digits and base five representation with four digits?

13. In the number base n, x is represented as 462_n. In the number base $2n$, x is represented as pqr_{2n} where p, q, r are integers between 0 and $2n-1$, inclusive. What is the value of $q + r$?

14. Let * be a binary operation defined by $a * b = \dfrac{a^2}{b - a}$. Find $(3*2)*-1$. Give your

    answer as a reduced fraction.

15. Let N be a two-digit number. If N is divided by its units digit the quotient is 6 and the remainder is 5. If N is divided by its tens digit the quotient is 11 and the remainder is 4. Find N.

16. The number 1234 is not divisible by 11 but the number 1342 is divisible by 11. How many different four digit numbers, each using all of the digits 1, 2, 3, and 4 are divisible by 11?

17. Find the largest four-digit perfect square that is divisible by 7 and the sum of whose digits is 25.

18. The Egyptians used unit fractions (fractions where the numerator is 1). They would express larger fractions as the sum of smaller unit fractions, i.e.

$$\frac{3}{4} = \bigcirc \bigcirc = \frac{1}{2} + \frac{1}{4}$$

    What fraction, written in reduced form, in the Hindu-Arabic system would be added to make the following statement a true one?

$$\frac{47}{60} = \bigcirc \bigcirc + ?$$

19. Determine the value of $\sqrt{\text{MCMXXXVI}}$. Write your answer in the Hindu-Arabic system.

20. Liz owned a flower shop and was having a sale on roses. She started the day with 463 roses to sell. A Roman came in and bought XCIV roses. An Egyptian came in and bought ∩⊙⊙|| roses. Base two man came in and bought 10101_{two} for his new friend. Base twelve man came in and bought a gross of roses. Base six man came in and bought all of the remaining roses. How many roses, in base six, did Base six man buy?