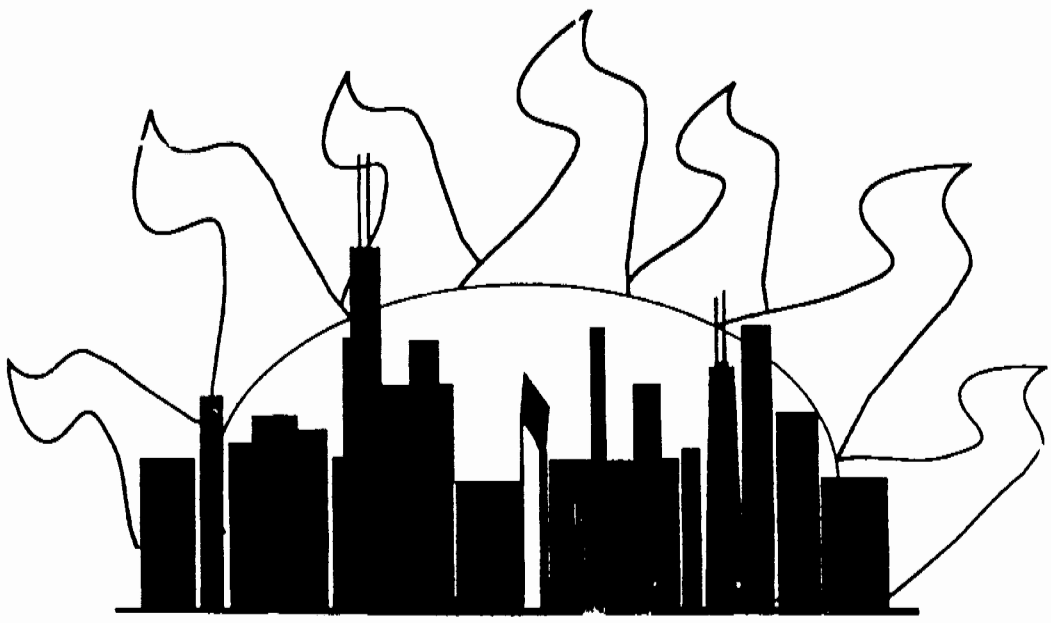
Theta Division **Topic Test 2**

Probability & Statistics



Mu Alpha Theta National Convention
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Theta Level

General Instructions:

Unless otherwise stated all answers should be written as decimals.
If you are asked to give your answer as a fraction, please give your answer in a / b form where a and b are relatively prime.

Questions

1. If n is a three-digit number whose units digit is a 3 and whose hundreds digit $\neq 0$, find the probability that n is divisible by 3. Give your answer as a reduced fraction.

2. Using the stem and leaf plot below, find the mean of the mean, median, and range of the following data.

9	0 2 3 5
8	1 2 6 7 9
7	1 4
6	2 6 7
5	3

Give your answer rounded to four significant digits.

3. On five tests (on which scores could range anywhere from 0 through 100), Johnny had an arithmetic mean of exactly 88. Find the lowest score Johnny could have received on one of these tests.

4. One stamp is randomly selected from a 10 by 10 sheet of 100 stamps. Find the probability that the stamp selected was not one of the border stamps. Write your answer as a reduced fraction.

5. A point is chosen at random from within a circular region. What is the probability that the point is closer to the center of the region than it is to the boundary of the region?

6. Five balls numbered 1 to 5 are in a bowl. Barbara reaches into the bowl, randomly removes one of the balls, and holds it. Nancy reaches into the bowl and randomly removes a different ball. What is the probability that the sum of the two numbers on the balls that were removed is an even number? Give your answer as a reduced fraction.

7. Three balls are marked 1, 2, 3. They are placed in a bowl and a ball is drawn, its number recorded, and the ball is returned to the bowl. The process is repeated two more times. If the sum of the three numbers is 6. what is the probability that the ball numbered 2 was drawn all three times? Give your answer as a reduced fraction.

8. A grocery store manager notes that 35% of customers who buy a particular product make use of a store coupon to receive a discount. If seven people purchase the product, what is the probability that fewer than four will use a coupon? Round your answer to three decimal places.

9. The names of the 50 states in the United States are placed in a bowl and one is drawn. What is the probability that the first letter of the name of the state is a vowel?

10. Two players flip three fair coins each. What is the probability that they get the same number of heads? Write your answer as a reduced fraction.

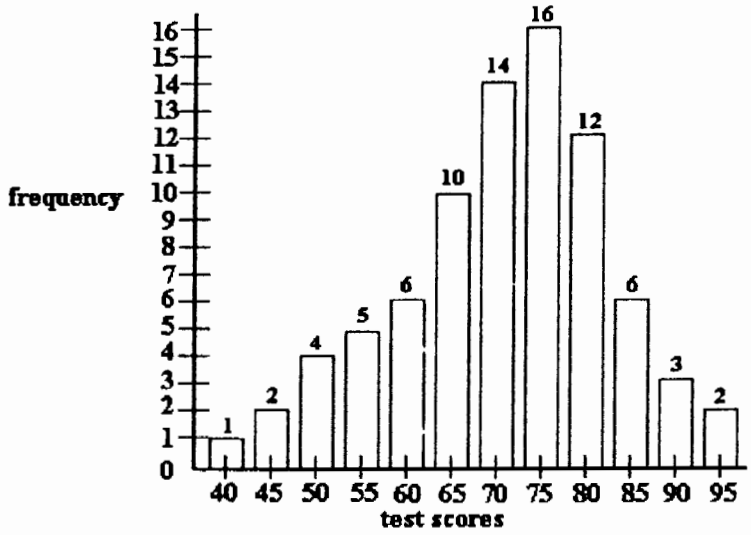
11. You have two six-sided dice with the numbers -1, 0, 1, 2, 3, 4 on each of them. When you roll the two dice, what is the probability that the product is greater than the sum? Give your answer as a reduced fraction.

12. At a dance, three couples are to be lined up for a picture. How many ways can they be lined up so that each person is next to his or her date?

13. A person is assumed to be of age n until his or her $n+1^{st}$ birthday. If two persons, each less than ten years old, are selected at random (i.e. their ages are between 0 and 9 inclusively and each of these ages is equally likely), what is the probability that the sum of their ages is at most 7? Give your answer as a reduced fraction.

14. The annual incomes of 100 families range from $8,200 to $98,000. In error, the largest income was entered on the calculator as $980,000. What is the difference, in dollars, between the mean of the incorrect data and the mean of the actual data?

15. Below is a histogram of the test scores of 81 students. Determine the mean. Round your answer to the nearest hundredth.



16. On a tiny, remote island a man can be granted mercy after solving a problem in the following way: He is given 18 white balls and 6 black balls. He must divide them among three boxes with at least one ball in each box. Then, blindfolded, he must choose a box at random, and then a ball from within this box. He receives mercy only if the chosen ball is white. What is the probability of the man receiving mercy provided he has distributed the balls in the most favorable manner? Write your answer as a reduced fraction.

17. A deck of 16 cards contains four aces, four kings, four queens, and four jacks. The 16 cards are thoroughly shuffled and my opponent, who always tells the truth, draws two cards simultaneously and at random from the deck. He says, "I hold at least one ace". What is the probability that he holds two aces? Give your answer as a reduced fraction.

18. If x and y are real numbers randomly chosen between 0 and 2, what is the probability that the hypotenuse of a right triangle with legs x and y will have a length less than 2? Give your answer as a reduced fraction.

19. A student was doodling randomly on a tic-tac-toe board. Find the probability that the first three boxes in which the student doodled correspond to a winning path of three marks in a straight line. Give your answer as a reduced fraction.

20. Suppose x is a real number. By how much does the median of
$x + 3,\ x - 2,\ x + 7,\ x + 5,\ x - 3,\ x + 2, x$
exceed then mean of these numbers? Give your answer as a simplified fraction.