Theta Ciphering Questions

P. For what values of K will the remainder of be the same when $x^3 - Kx + 3x - 5$ is divided by $x - 2$ and $x + 3$.

1. Find $x + y + z$ given that: $\log_2 (\log_3 (\log_4 x)) = 0$
$\log_3 (\log_2 (\log_4 y)) = 0$
$\log_4 (\log_3 (\log_2 z)) = 0$

2. Find the smallest root of the equation $(x+2)(x+5)(x+9) - (x+2)(x+5)(2x+13) = 0$.

3. A hexagon is inscribed in a circle. Three of the sides are of length 3 and the other three sides are of length 4 and no two adjoining sides are the same length. What is the radius of the circle?

4. Find $A^2 - B^2$ if $A = 2^{2003} + 2^{-2003}$ and $B = 2^{2003} - 2^{-2003}$.

5. How many linear arrangements (i.e. listings) of the six digits 1, 2, 3, 4, 5, 6, using each once, either start with a 2 or end with a 5 or both?

6. If the points (0, 0), (5, 3) and (8,0) are points on the circumference of a circle, determine the area of the circle.

7. Simplify by rationalizing the denominator: $\dfrac{1}{1+\sqrt{2}+\sqrt{3}}$

8. A water tank is in the shape of an inverted right circular cone (vertex down). The radius of its base is 16 feet, and its height is 96 feet. What is the height, in feet, of the water in the tank if the amount of water is 25% of the tank's capacity?

9. If $2 \log_3(x-2y) = \log_3 x + \log_3 y$, find $\dfrac{x}{y}$.

10. Two positive integers have a product of one billion and neither integer contains a zero. Find the smaller integer.