EQUATIONS AND INEQUALITIES – THETA

Mu Alpha Theta National Convention 2003

For all questions, answer E "NOTA" means none of the above answers is correct

1. Find the extraneous root of the following equation: $\log_{10} a + \log_{10}(a+21) = 2$

   A) 4 B) -25 C) -21 D) -4 E) NOTA

2. If the points $(0,-5), (-1,-7)$ and $(-4,-1)$ are on the graph of $y = ax^2 + bx + c$, find abc.

   A) 0 B) -15 C) 15 D) 45 E) NOTA

3. For what positive values of k does $2x^2 + kx + 3k = 0$ have only one real solution?

   A) $0, 24$ B) $2\sqrt{6}$ C) 24 D) 6 E) NOTA

4. The equation of an ellipse is $x^2 + 9y^2 - 4x + 54y + 49 = 0$.
   Find the product of the ordinate of the center and the length of the major axis.

   A) 24 B) -108 C) -36 D) -18 E) NOTA

5. Find the coefficient of the linear term of the polynomial function of least degree with integral coefficients whose zeros are 6 and $4 - 2i$.

   A) 68 B) -2 C) 24 D) -12 E) NOTA

6. Find xy: $\begin{bmatrix} 5 & 3 \\ 7 & 5 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ -11 \end{bmatrix}$

   A) $\frac{11}{7}$ B) $\frac{11}{105}$ C) -10 D) 0 E) NOTA

7. If $\begin{vmatrix} x^2 & x \\ -2 & 5 \end{vmatrix} = 3$, then what is the largest value of x that satisfies this equation?

   A) 1 B) $\frac{3}{5}$ C) -1 D) $\varnothing$ E) NOTA

8. Solve the system $\begin{aligned} x + 2z &= 5 \\ -4x + 6y &= 0 \\ x - 2z &= 9 \end{aligned}$ and find the value of $\frac{x}{y} - 5z$.

   A) $-\frac{7}{2}$ B) $\frac{13}{2}$ C) $\frac{17}{3}$ D) $-\frac{13}{3}$ E) NOTA

9. If your investment of $2,000 earns interest compounded continuously and is worth $2,800 after four years, how much will it be worth after 16 years?

A) about $9,820 C) about $7,680 E) NOTA

B) about $8,400 D) about $3,200

10. In a system of 3 equations with 3 variables, with each equation in the form $ax + by + cz = d$, Cramer's rule was used to solve for x by using:

$$x = \frac{\begin{vmatrix} 5 & -1 & -1 \\ 7 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & -2 & -3 \end{vmatrix}}.$$

What is the value of y?

A) -2 B) -1 C) 0 D) 2 E) NOTA

11. Find the function of least degree with leading coefficient of 1 and whose graph has x-intercepts of $(-2,0)$, $(-3,0)$ and $(1,0)$.

A) $f(x) = x^3 + 6$ D) $f(x) = x^3 + 4x^2 - x + 6$

B) $f(x) = x^3 - 4x^2 - x + 6$ E) NOTA

C) $f(x) = x^3 + 4x^2 + x - 6$

12. If $2 \cdot 9^x + 15^x = 25^x$ and $x = \log_{3/5} m$, find the numerical value of $-\frac{2}{m}$.

A) $\frac{3}{5}$ B) -3 C) -4 D) $-\frac{10}{3}$ E) NOTA

13. Find the value of the discriminant of the quadratic equation that has one solution of (2+3i) with leading coefficient of 1.

A) -13 B) -9 C) -36 D) -5 E) NOTA

14. If the solution to the equation $8^{x-2} = 5^x$ is rounded to the nearest thousandth, what digit is in the hundredth's place?

A) 8 B) 4 C) 5 D) 9 E) NOTA

15. Find the equation of the circle that is tangent to $x=-3$, $x=5$, and the x-axis, if the center of the circle is in the first quadrant.

A) $x^2+6x+y^2+12y+45=0$

B) $x^2-2x+y^2-8y+33=0$

C) $x^2-4x+y^2-8y+13=0$

D) $x^2-2x+y^2-8y+1=0$

E) NOTA

16. If $15^{2x+3}=27^x\cdot 5^{y+2}$, find y where $-10<y<10$.

A) 9 B) -5 C) -1 D) $\varnothing$ E) NOTA

17. If $\frac{A}{x+4}+\frac{B}{x-7}=\frac{11x-22}{x^2-3x-28}$, find the value of $3A-2B$.

A) -52 B) 28 C) -12 D) 8 E) NOTA

18. Find the sum of the roots of $|x-9|=4x+3$.

A) $\frac{6}{5}$ B) $-\frac{14}{5}$ C) $\frac{8}{5}$ D) $-\frac{26}{5}$ E) NOTA

19. If $a^3-b^3=24$ and $a-b=2$, find $a+b$.

A) $\pm\frac{2}{3}\sqrt{11}$ B) $\pm\frac{2}{3}\sqrt{33}$ C) $\pm\frac{2}{3}\sqrt{39}$ D) $\pm\frac{2}{3}\sqrt{21}$ E) NOTA

20. Solve for x: $7-\frac{1}{7^{-1}+x^{-1}}=\frac{1}{7}$

A) 7 B) 28 C) 56 D) 336 E) NOTA

21. What is the value of $x-y$ for the ordered pair (x,y) that is the solution to the system:

$$\frac{x+2y}{xy}=\frac{11}{12} \qquad \frac{2x-3y}{xy}=\frac{2}{3}$$

A) $\frac{186}{119}$ B) $\frac{30}{7}$ C) $\frac{196}{119}$ D) -7 E) NOTA

22. Solve over $\Re$: $3\{2x-[10-3(2x+3)]\}=5(3x-4)-(5x-2)$.

A) $-\frac{15}{14}$ B) $-\frac{19}{14}$ C) $-\frac{45}{46}$ D) $-\frac{15}{22}$ E) NOTA

23. Given that $f(x)=2x+1$, find the sum of the values of x for which it is true that $f(x+1)=\frac{12}{f(x)-2}$.

A) -1 B) -4 C) 1 D) 4 E) NOTA

24. If $x^3-12x^2+41x-42>0$, which of the following could be a value for x^2+1?

A) 1 B) 5 C) 37 D) 51 E) NOTA

25. Let c be the complex root of $f(x)=x^3-1$, such that $c=a+bi$ and a and b are both negative. Find the value of b^2.

A) .75 B) 1 C) 1.5 D) 3 E) NOTA

26. Find the absolute value of the difference of the roots of $(x-\frac{4}{9})(x-\frac{4}{9})+(x-\frac{4}{9})(x-\frac{1}{3})=0$.

A) $\frac{1}{3}$ B) $\frac{4}{9}$ C) $-\frac{1}{3}$ D) $-\frac{4}{9}$ E) NOTA

27. If the solution to the equation $x-4=x\sqrt{3}$ is expressed as $a+b\sqrt{c}$ and all fractions are simplified, then give the value of a.

A) -2 B) -1 C) 1 D) 2 E) NOTA

28) What is the area of the region bounded by the equation $|x|+|y|\le 3$?

A) 4.5 B) 9 C) 18 D) $18\sqrt{2}$ E) NOTA

29) How many non-positive integral solutions exist for the inequality $|x+2|\le 2x+7$?

A) 3 B) 4 C) 5 D) 6 E) NOTA

30) For the system $x + y \ge 2$
$4y \le x + 8$
$2y \ge 3x - 6$ what is the minimum value of the function f(x,y) = 3y + x ?

A) -34 B) -6 C) -2 D) 2 E) NOTA