Theta Gemini

Mu Alpha Theta National Convention 2003

The abbreviation "NOTA" denotes "None of These Answers." All diagrams provided lie in one plane.

1. An isosceles right triangle has area 100 square inches. Give the length of its hypotenuse.

A. 10 in. B. $10\sqrt{2}$ in.
C. 20 in. D. $50\sqrt{2}$ in. E. NOTA

2. Give the next term in the sequence 1, 2, 3, 5, 8, 13, ___.

A. 16 B. 20
C. 21 D. 24 E. NOTA

3. If $\sqrt{8+2\sqrt{15}} = \sqrt{A}+\sqrt{B}$ then for $A > B$, give the value of $A - B =$ __.

A. 1 B. 2 C. 3 D. 4 E. NOTA

4. The sum of the reciprocals of A and B is $\frac{5}{8}$ and the product of A and B is 4. What is the sum of A and B?

A. $\frac{1}{4}$ B. $\frac{5}{4}$
C. $\frac{5}{2}$ D. $\frac{8}{5}$ E. NOTA

5. If $(x-1)(2x+3) = 9$ then what is the value of $x(2x+1)$?

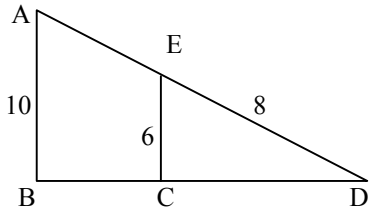
A. 6 B. 9
C. 12 D. 27 E. NOTA

6. If $\frac{1}{\sqrt{\frac{1}{x}-\frac{1}{2}}} = 8$ and $\sqrt{y}+2 = \sqrt{33x}$ then give the value of y.

A. $\frac{3\sqrt{6}}{4}$ B. $\sqrt{6}$
C. $12-8\sqrt{2}$ D. 36 E. NOTA

7. $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{EC}$ and $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{BD}$. If AB=10, EC=6, and ED=8, then give the length BC.

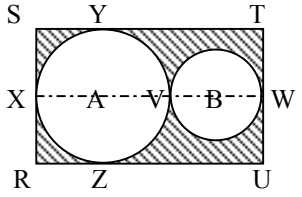
A. $1.\overline{3}$
B. $1.\overline{6}$
C. $1.\overline{3}\sqrt{7}$
D. $6.\overline{6}$
E. NOTA



may not be drawn to scale

8. Circle A is tangent to rectangle RSTU at three points (X, Y, Z) as shown, and circle B is tangent to circle A at V and also to the rectangle (at W) as shown. X, A, B, V and W are collinear. RS=6, ST=10. What is the area outside of the two circles and inside of the rectangle (shaded)?

A. 48π
B. $60-13\pi$
C. $60-12\pi$
D. $60-11\pi$
E. NOTA



9. Line $\overleftrightarrow{AB}$ has slope -3 and y-intercept 23. Line $\overleftrightarrow{AC}$ has x-intercept -4 and y-intercept 2. If intersection point A has coordinates (m, n) then give the value of m+n.

A. 9 B. 10
C. 11 D. 12 E. NOTA

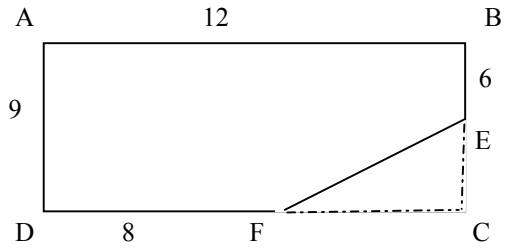
10. The circumference of a circle is $\frac{5}{2}\pi$ and its center is a distance of 5 from point P, in the same plane. Point Q lies on the circle. Which could NOT be the distance PQ ?

A. 3.75 B. 5.00
C. 6.02 D. 6.25
E. NOTA

11. If $x+y+z=6$, $3x-y+3z=6$ and $x-y+z=0$ then what is the value of $x+z$?

A. 2 B. 3
C. 4 D. 6 E. NOTA

12. Rectangle ABCD with dimensions 9 by 12 bounds a region. A triangular region (EFC) was cut off the corner of this region, as shown. What is the perimeter of the resultant polygonal region (ABEFD)?

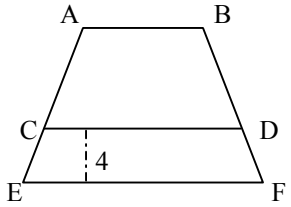


A. 40 B. 38
C. 37 D. 36 E. NOTA

13. $\overline{CD}$ is parallel to bases $\overline{AB}$ and $\overline{EF}$. The area of ABFE is 72, and $\overline{CD}$ is a distance of 4 from $\overline{EF}$. If AB= 6 and EF= 10 and the area of trapezoid ABDC is $35.\overline{5}$ then give the length CD.

A. 8 B. $\frac{125}{3}$

C. $\frac{74}{9}$ D. $\frac{35}{6}$



not drawn to scale

E. NOTA

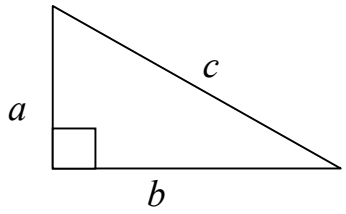
14. If n is an even integer greater than 2, and n_2 is the next (first) greater even integer, and n_3 is the next (second) greater even integer, and so on, then which is an expression in terms of n for the 100th next greater even integer (after n) ?

A. $n+198$ B. n^{50}
C. $n+50$ D. $n+200$ E. NOTA

15. Consider the set S= {2, 3, 5, 6, 7, 10, 12 } How many different pairs of unequal numbers can be chosen from set S so that their sum is greater than 12 ? (The pair {2,3} is considered the same as the pair {3,2}.)

A. 18 B. 13
C. 12 D. 11 E. NOTA

16. If $\frac{a}{b} = \frac{2}{\frac{1}{3}+\frac{1}{4}}$ and $\log c = 2$ then give the perimeter of the right triangle shown.



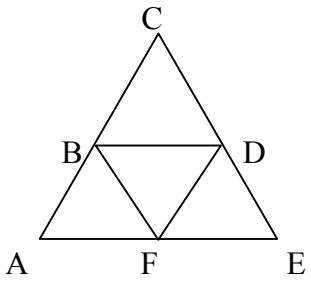
A. 56
B. 112
C. 224
D. 1344

17. If $x^2 = 25$ and $y^2 = 100$ then give the greatest possible value of $y - x$.

A. 15 B. 10
C. 5 D. -5 E. NOTA

18. How many distinct quadrilaterals can be found in the diagram shown. Each quadrilateral has vertices among the set {A, B, C, D, E, F} . (Note: Figures such as ABCD are not considered a quadrilateral.)

A. 15
B. 9
C. 6
D. 4
E. NOTA

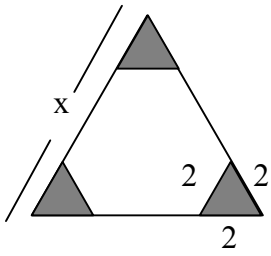


19. A piece of wire of length x feet, y inches ($y < 12$) is cut into exactly 100 pieces, each 2 feet and 4 inches in length. What is the value of $x + y$?

A. 50 B. 233
C. 236 D. 237 E. NOTA

20. An equilateral triangle has sides of length x. Each vertex is cut from the region bounded by the equilateral triangle so that a smaller equilateral triangle region of side length 2 is removed, as shown. (The shaded regions shown are removed.) The remaining hexagonal region has area equal to thirteen-sixteenths of the area of the original equilateral triangle. Give the value of x.

A. $13\sqrt{3}$
B. 13
C. $8\sqrt{3}$
D. 8
E. NOTA



21. Thirty students made their college schedules for the next term. 8 took math, 14 took science, 17 took literature, 7 took both science and literature, 3 took math and literature, and 3 took math and science. If 2 took all three (math, science, and literature), then how many took none of the three courses (neither math, science, nor literature)?

A. 2 B. 3
C. 5 D. 19 E. NOTA

22. If $\frac{1}{\sqrt[3]{2}+\sqrt[3]{3}} = \frac{\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}}{d}$ for $|c| > |b| > |a|$, c and d are relatively prime, then $c + d$ = ?

A. 5 B. 8
C. 9 D. 14 E. NOTA

23. If $(a+b)^2 = 20$ and $a^2 + b^2 = 16$ then give the value of ab.

A. 4 B. 2
C. 1.25 D. 1 E. NOTA

24. If $\sqrt{20-\sqrt{20-\sqrt{20-\sqrt{...}}}} = x$ then $x =$

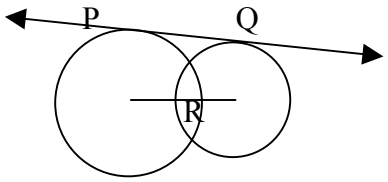
A. 6 B. 5
C. $2\sqrt{5}$ D. 4 E. NOTA

25. Using two types of regular polygons only, these polygons tessellate about a point P. Which could be these polygons?

A. triangle, pentagon B. square, octagon
C. hexagon, pentagon D. square, hexagon
E. NOTA

26. Two circles have radii 4 and 9. A common tangent is drawn, with P and Q points of tangency as shown. The centers of the circles are joined and the segment meets the smaller circle at point R. If $\overline{PQ}$ has length $5\sqrt{3}$, then give the length of arc RQ. P is a point on the larger circle.

A. $\frac{4}{3}\pi$
B. $\frac{8}{3}\pi$
C. $\frac{16}{3}\pi$
D. $\frac{20}{3}\pi$ E. NOTA



27. Two circles, A and B, of radii 10 intersect at points P and Q. If AB=12, then what is the length of $\overline{PQ}$? (A and B are centers of the circles.)

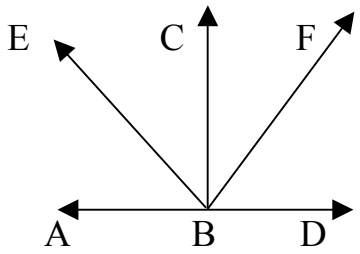
A. 10 B. 12 C. 14 D. 16 E. NOTA

28. Pythagorean Triples may be generated by u^2+v^2, $2uv$, and u^2-v^2, for u and v distinct positive integers. Give the sum of the hypotenuses of all right triangles with integral length sides, with one leg of length 16.

A. 54 B. 85 C. 119
D. 143 E. NOTA

29. $\overline{CB} \perp \overline{AD}$ and $\overrightarrow{BC}$ bisects $\angle EBF$. The diagram is not drawn to scale. $m\angle EBC = 5x-36$, $m\angle CBF = 4x-7y$ and $m\angle FBD = 3x+2y$. Give the value of $x+y$. (All angle measures are in degrees.)

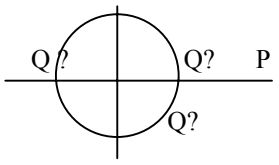
A. 18
B. 20
C. 39
D. 51
E. NOTA



30. In circle A, $\overline{CD}$ is a diameter of the circle. Point E lies on the circle, and is not collinear with C and D. If CD=20 and $m\angle ECD = 30^\circ$ then give the area outside of ΔECD and inside of the circle.

A. $100\pi - 100\sqrt{3}$
B. $100\pi - 50\sqrt{3}$
C. $400\pi - 100\sqrt{3}$
D. $400\pi - 50\sqrt{3}$
E. NOTA

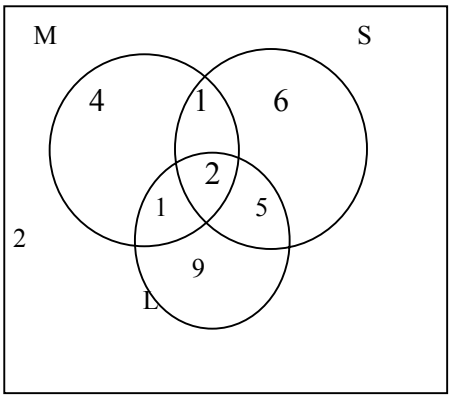
Solutions:

1. $\frac{1}{2}x^2 = 100,\ x = \sqrt{200}$. Hypotenuse=20, choice C.
2. Add two previous terms. Choice C.
3. Square each side: $8 + 2\sqrt{15} = A + 2\sqrt{AB} + B$ so A+B=8 and AB=15. A, B are 3 and 5. so A-B=2, choice B.
4. Get a common denominator: $\frac{A+B}{AB} = \frac{5}{8}$ and $\frac{A+B}{4} = \frac{5}{8}$ so A+B=2.5. Choice C.
5. Expand and simplify to $2x^2 + x - 12 = 0$. which gives $2x^2 + x = 12$ and so 2x(x+1) is 12, choice C.
6. Reciprocate: $\sqrt{\frac{1}{x} - \frac{1}{2}} = \frac{1}{8}$ so $\frac{1}{x} - \frac{1}{2} = \frac{1}{64}$. Add 1/2 from each side, and reciprocate again to get x=64/33. In the 2nd equation, $\sqrt{y} + 2 = \sqrt{64}$ and so $\sqrt{y} = 6,\ y = 36$. Choice D.
7. Use the Pythagorean Th. to find CD= $2\sqrt{7}$ and then $\frac{6}{10} = \frac{CD}{BD}$. and substituting CD and solving, we get $BD = \frac{10}{3}\sqrt{7}$. So BC= $\frac{10}{3}\sqrt{7} - 2\sqrt{7} = \frac{4}{3}\sqrt{7}$, choice C.
8. The big circle has area 9π so the little circle's radius is 2. The area of that is 4π. Area of rectangle minus circles is $60 - 13\pi$. Choice B.
9. AB: y= -3x+23 or 3x+y=23. AC: 2x-4y= -8. Mult. the top equation by 4 and add to get 14x=84, so x=6 and thus y=5. Choice C for the sum of 11.
10. 

Set P and the center of the circle on the x-axis. PQ can range from 5-5/4 to 5+5/4. The radius of the circle is 5/4 (from $C = 2\pi r$), or from 3.75 to 6.25. Choice E, since all are possible.
11. Add the first and last equations to get 2x+2z=6 so x+z=3. Choice B.
12. The slanted segment is 5 , by the Pyth. Th. so P=12+6+5+8+9 = 40. Choice A.
13. 100 = median(h), so $100 = 0.5(6+10)h$ so h=9 and the area of ABDC=320/9 so 320/9=0.5(5)(6+x) which gives x=74/9 which is choice C.
14. The first next even term is n+2, and the second is n+4, ... the fifth is n+8, and in general, the rth is n+(r-1)2. So the 101th is n+200. Choice D.
15. Total of 11: 2,12. 3, 10. 3,12. 5,10. 6,12. 6,7. 5,10. 6,12. 7,10. 7,12. 10,12. Choice D.
16. $\frac{24}{7} = \frac{a}{b}$ and since c=100, this is a triple 4(7-24-25) = (28-96-100). Perimeter=224. Choice C.
17. y-x is maximized if y=10 and x= -5 and y-x = 15. Choice A.
18. Six: ABDE, ABDF, BDEF, ACDF, BCDF, BCEF. Choice C.
19. 200 feet and 400 inches gives 233 feet 4 inches which gives a sum of 237. Choice D.

20. Each little triangle shaded has an area of $\frac{side^2}{4}\sqrt{3}$ which is $\sqrt{3}$ each. So $3(\sqrt{3}) = \frac{3}{16}\left(\frac{x^2}{4}\sqrt{3}\right)$ which gives x=8. Choice D.

21. In the diagram to the right we see that 2 elements lie outside of the 3 circles. Choice A.



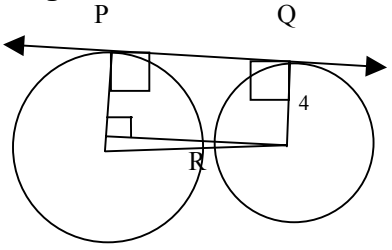
22. Using the sum of cubes formula multiply numerator and denominator by $\sqrt[3]{4} - \sqrt[3]{6} + \sqrt[3]{9}$ to get a=4, b= -6, c=9, d=5. and c+d=14, choice D.

23. Expand: $a^2 + 2ab + b^2 = 20,\ 2ab = 20 - 16 = 4,\ ab = 2$ which is choice B.

24. $x^2 = 20 - x$ and $x^2 + x - 20 = 0$ solves to x=4 or x= -5. The positive answer gives 4, choice D.

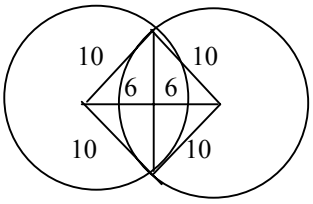
25. Some combination of interior angles must add to 360 degrees. In an octagon we can have two octagons and one quad, which gives 135+135+90. Choice B.

26.



In the diagram below left, we have a small right triangle formed by connecting the centers, and the radii, and the lower side of the rectangle shown. Since the small leg is 5, and the other leg is $5\sqrt{3}$, the central angle for RQ is 30+90 or 120 deg. So $\frac{120}{360}(2 \bullet \pi \bullet 4) = \frac{8}{3}\pi$. Choice B.

27.



The rhombus formed has diagonal 12, and diagonals form 4 6-8-10 triangles. So PQ is 16. Choice D.

28. Legs are 2uv or $u^2 - v^2$ so we have either 2uv=16 or $u^2 - v^2 = 16$. The first gives uv=8 so we may have {u,v}={8,1} or [4,2}. This gives 16,63,65 and 12,16, 20. In the second, (u-v)(u+v)=16 and so u-v=2 and u+v=8 which adds to give u=5,v=3 or the triple 16, 30, 34. If u-v=1 and u+v=16, we get no triple. So the sum of the hypotenuses is 65+20+34=119. Choice C.

29. $5x - 36 = 4x - 7y,\ (4x - 7y) + (3x + 2y) = 90$ The system then solves to y=3 and x=15. The sum is 18, choice A.

30. A triangle inscribed in a semicircle is a right triangle. So ED=10, CE=$10\sqrt{3}$ which gives an area of $\frac{1}{2}bh = \frac{1}{2}(10)(10\sqrt{3}) = 50\sqrt{3}$. The circle has radius 10 which gives a circle of area 100π so the area requested is $100\pi - 50\sqrt{3}$, choice B.