Theta Individual Test

Mu Alpha Theta National Convention 2003

The abbreviation NOTA denotes "None Of These Answers."

1. For what value of k is $x = 5$ a solution to the equation $x^2 + 3 = kx + 9$?

A. $\frac{19}{5}$ B. $\frac{13}{5}$

C. $\frac{4}{5}$ D. $\frac{1}{5}$ E. NOTA

2. If w is increased by 400% of w and the result is decreased by 80% (of the result) then 54 is the final value. What is 80% of w ?

A. 13.5 B. 43.2
C. 54.0 D. 67.5 E. NOTA

3. If $\frac{1}{2}A + \frac{1}{3}B = \frac{1}{5}$ and $A = 2B$, then what is the value of $100(A + B)$?

A. 18 B. 45
C. 60 D. 63 E. NOTA

4. The sum of two numbers is 2 and the difference of the same two numbers is 4. What is the sum of the cubes of the same two numbers?

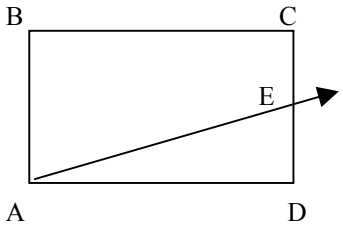
A. 26 B. 28
C. 56 D. 72 E. NOTA

5. A line with slope 4 and x-intercept -8 contains the point P which is 40 units above the x-axis. What is the x-coordinate of point P ?

A. 2 B. 16
C. 18 D. 32 E. NOTA

6. Rectangle ABCD is drawn with AD=10 and BA=6. Line $\overleftrightarrow{AE}$ is drawn with slope $\frac{1}{5}$, and $\overleftrightarrow{AD}$ has slope zero. Point E is on $\overline{CD}$. Give the length AE.

A. $2\sqrt{26}$
B. $2\sqrt{29}$
C. $\sqrt{101}$
D. $\sqrt{401}$
E. NOTA

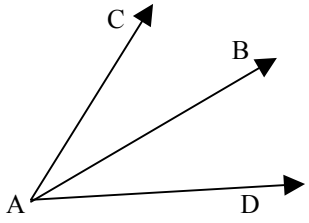


7. For $a = b + 1$, and $ab \neq 0$ the expression $-\left(\frac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a} - \frac{1}{b}}\right)$ is equal to which of the following?

A. $2b + 1$ B. $2b - 1$
C. $-2b + 1$ D. $-2b - 1$ E. NOTA

8. $\overrightarrow{AB}$ bisects $\angle CAD$. If $m\angle CAB = 4x - 8$ and $m\angle BAD = 2y + x$ and $m\angle CAD = 100^\circ$ then find the value of y.

A. 11.50
B. 14.50
C. 17.50
D. 17.75
E. NOTA

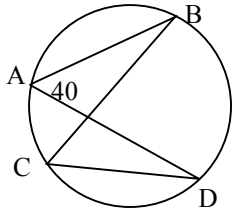


9. Lines *l*, *m* and *n* are three distinct lines. Line *l* is perpendicular to line *m*, and line *m* is perpendicular to line *n*. Lines *l* and *n* may be ...

i) parallel ii) perpendicular iii) skew

A. i only B. ii only
C. i, iii only D. ii, iii only E. NOTA

10. $m\angle BAD$, inscribed in the circle as shown, is 40°. The measure of arc CA is 60°. Give the measure of $\angle ADC$.



A. 30°
B. 40°
C. 60°
D. 80°
E. NOTA

11. A right triangle has sides of lengths 6, 8 and $4x+2$. Which could be the value of x?

A. $\sqrt{7}-\frac{1}{2}$ B. $\frac{\sqrt{7}}{4}$
C. $\frac{3}{2}$ D. $\frac{\sqrt{7}}{2}-\frac{1}{2}$
E. NOTA

12. A rhombus has perimeter 40 and one diagonal of length 12. Give the area of the rhombus.

A. 48 B. 96
C. 120 D. 240 E. NOTA

13. If x and y are positive integers and $x+y<150$, and $x>10$, then what is the least possible value for $x-y$?

A. -139 B. -128
C. -127 D. -125 E. NOTA

14. An isosceles triangle has legs each of length 4. The altitude to the base has length 3. Give the length of the base.

A. 5 B. $2\sqrt{7}$
C. $4\sqrt{3}$ D. 10 E. NOTA

15. If $\log_2 A=\log_8 B$ and $\log_4 B=C$, and $\log A=\log(2^K)$ then give the value of k.

A. $\frac{3}{2C}$ B. $\frac{3C}{2}$
C. $\frac{2}{3C}$ D. $\frac{2C}{3}$ E. NOTA

16. If $\frac{\sqrt{2}}{\sqrt{2}-3}=A+B\sqrt{C}$ where $B\sqrt{C}$ is in simplest radical form, then which is the value of $B-A$?

A. -1 B. $\frac{-1}{7}$
C. $\frac{1}{7}$ D. 1 E. NOTA

17. Line L, perpendicular to the line with equation $y=3x-5$, contains the point (1, 4). What is the x-intercept of L?

A. 12 B. 13
C. 14 D. 15 E. NOTA

18. If r is a solution to the equation $(x-1)(x+3)=5$ then $4r+1$ could be ...

A. 25 B. 16
C. 7 D. 9 E. NOTA

19. The number 401_{five}, written in base-five, is equal to the expression $100A+A$, a base ten expression. Give the value of A.

A. 1 B. 2
C. 3 D. 4 E. NOTA

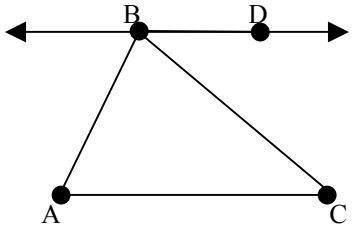
20. You have 42 inches of string. You want to cut it to form an equilateral triangle and a square, using all of the string for both. If the sides of the equilateral triangle and the square are congruent and distinct, what is the area of the square, in square inches?

A. 24 B. 36
C. 49 D. 70.56 E. NOTA

21. A team played its first 20 games and won 15 of them. No ties are allowed. They then played five more consecutive games and lost all five. What is the minimum number of consecutive games that this team must now play and win to bring its winning percentage back to at least what it was before the five game losing streak?

A. 2 B. 12
C. 15 D. 17 E. NOTA

22. Lines $\overleftrightarrow{BD}$ and $\overleftrightarrow{AC}$ are parallel, and AC=10. If $m\angle DAC = 30^\circ, m\angle BAC = 60^\circ$ and the area of ΔABC is 75, which is the area of ΔDAC?

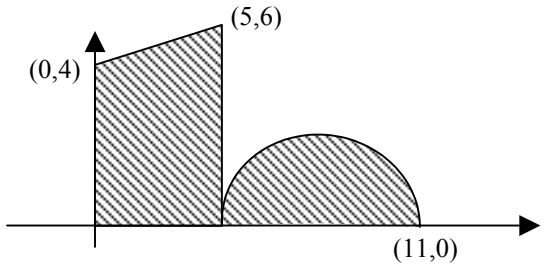


A. 37.5
B. $37.5\sqrt{2}$
C. $37.5\sqrt{3}$
D. 75
E. NOTA

23. The graph of $9x^2 + 25y^2 = 225$ has foci at points P and Q. The graph of $x^2 + y^2 - 8x + 4y - 2 = 0$ has center R. Of the distance PR, and QR, give the distance that is the larger.

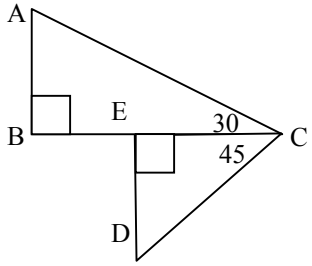
A. 2 B. 6
C. $2\sqrt{17}$ D. $2\sqrt{34}$ E. NOTA

24. Let A be the total combined area of the shaded region, which consists of the area of a trapezoid and a semicircle. The parallel bases of the trapezoid are vertical. If $2A = B + C\pi$ then $B + C =$



A. 29.5 B. 43
C 59 D. 109 E. NOTA

25. In the diagram shown, AC=12 and BE=5, $m\angle ACB = 30^\circ$ and $m\angle ECD = 45^\circ$. B, E and C are collinear. Give the value of CD.

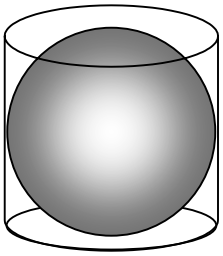


A. $\sqrt{2}$
B. $6\sqrt{6} - 5\sqrt{2}$
C. $3\sqrt{6} - 2.5\sqrt{2}$
D. $6\sqrt{6} - 5$
E. NOTA

26. The graph of a parabola opens downward, with y-intercept 10 and x-intercepts -1 and 5. If the point P(8, k) lies on the graph of the parabola, what is the value of k?

A. -60 B. -54
C. -27 D. -8 E. NOTA

27. A sphere of radius R inches is inscribed in the cylinder shown so that the sides and bases of the cylinder touch the sphere. The volume of the region outside of the sphere and inside the cylinder is 18π cubic inches. Give the area of the great circle of the sphere.



A. 36π B. 16π
C. 9π D. 5.76π
E. NOTA

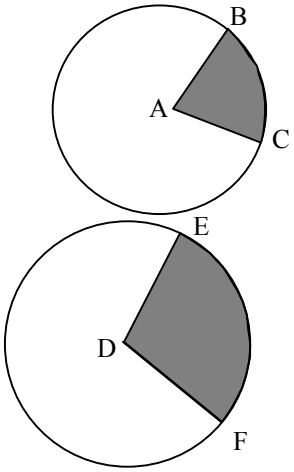
28. X liters of a mixture with 20% acid are mixed with four liters of pure acid, to result a new mixture that is 25% acid. How many total liters (X) were in the original solution before the pure acid was added?

A. 15 L B. 24 L

C. 60 L D. 80 L E . NOTA

29. Arc BC has a degree measure that is $\frac{3}{4}$ that of arc EF, shown. The area of the sector shaded in circle A, bounded by arc BC , is $\frac{1}{8}$ that of the sector (shaded) in circle D, bounded by arc EF. If $\overline{AB}$ has length x then what is the length of $\overline{DE}$ in terms of x ? (diagram not drawn to scale)

A. $48x$
B. $4\sqrt{3}x$
C. $\sqrt{6}x$
D. $\frac{4\sqrt{2}}{3}x$
E. NOTA



30. A triangle has angles of measures $(9x+9)$ degrees, $(4x+4)$ degrees, and kx degrees, for some constant k. Each angle is between 0 and 180 degrees. If the triangle is a right triangle, then what is the largest possible value for k ?

A. $\frac{77}{13}$

B. 10

C. $\frac{1170}{77}$

D. There is no upper bound for k.

E. NOTA

Solutions:

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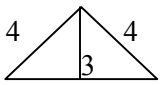
1. $25+3=5k+9.\quad k=19/5.$ Choice A.
2. $(w+4w)0.20=54.\quad w=54.\ \ 0.80w=43.2$ Choice B.
3. Multiply the first equation by 30. 15A+10B=6. Substitute to get 30B+10B=6, B=3/20 and A=3/10. 100 times the sum is 45. Choice B.
4. x+y=2 and x-y=4. Adding gives 2x=6. x=3 and y= -1. Sum of cubes is 27+ -1 = 26. Choice A.
5. Using slope 4, and point (-8,0) gives y-0=4(x+8). Substituting (k,40), 40=4(x+8) and x=2. Choice A.
6. $\frac{1}{5}=\frac{ED}{AD},\ \frac{1}{5}=\frac{ED}{10}$ and ED=2. Using the Pythagorean Theorem we get AE= $\sqrt{100+4}$ which is $2\sqrt{26}$ or choice A.
7. Multiply numerator and denominator by ab gives $-\frac{a+b}{b-a}$ and substituting gives $\frac{b+1+b}{b+1-b}=2b+1$ which is choice A.
8. 4x-8=50, x=14.5, and 2y+14.5=50 so y=17.75. Choice D.
9. Since the lines are not necessarily coplanar choices i, ii and iii can be true. The lines may be perpendicular if all three lines intersect at one point. Choice E.
10. Since arc AC is 60, angles ABC and ADC are both 30 degrees. Choice A.
11. If 4x+2 is the hypotenuse then 4x+2=10 and x=2, not a choice. If 8 is the hypotenuse, then $4x+2=\sqrt{64-36}=2\sqrt{7}$ so x= $\frac{2\sqrt{7}-2}{4}=\frac{\sqrt{7}-1}{2}$ which is choice D.
12. The sides are each 10 and diagonals are perpendicular so we have four 6-8-10 right triangles in the interior. So each has area $\frac{1}{2}(6)(8)$ which is 24. And 24X4=96. Choice B.
13. x may be between 10 and 149. To minimize x-y, and maximize y we use 11-138 for the difference. This is -127, choice C.
14. Using the Pythagorean Th. gives half the base is $\sqrt{7}$ so the base is $2\sqrt{7}$, choice B.



15. Using the change of base rule, $\frac{\log A}{\log 2}=\frac{\log B}{3\log 2}$ which gives $3\log A=\log B$. In the second equation, $\frac{\log B}{2\log 2}=C$ and thus $\frac{3\log A}{2\log 2}=C$ so $\log A=\frac{2\log 2}{3}\cdot C=\log 2^{\frac{2C}{3}}$ which gives a power of 2C/3, choice D.
16. Multiplying numerator and denominator by the conjugate $\sqrt{2}+3$ gives $\frac{2+3\sqrt{2}}{-7}$ so B= -3/7 and A= -2/7, and B-A= -1/7. Choice B
17. The slope of the perpendicular line is -1/3. The equation is then x+3y=13, and if y=0, x=13. Choice B.
18. $x^2+2x-3=5,\ x^2+2x-8=0,$ (x+4)(x-2)=0 gives x = r = -4 or 2. So 4r+1= -15 or 9. Choice D.
19. 4(25)+0(5)+1=101. 101A=101 gives A=1. Choice A.
20. Seven sides for 42 inches gives 6 inches for each. Area of the square is then 36. Choice B.
21. $\frac{15+A}{25+A}=\frac{3}{4}$ (which is 75%) gives A=15. Choice C.
22. Same base and same height gives the same area. The angles are irrelevant. Choice D.

23. $\frac{x^2}{25}+\frac{y^2}{9}=1$ gives a=5 and b=3 and $a^2-b^2=c^2$ and c is the distance from center to focus. So foci are (4,0) and (-4,0). Completing the square for the second conic gives $(x-4)^2+(y+2)^2=22$ so center R is at (4, -2). The distances PR and PQ are then 2 and $\sqrt{8^2+2^2}=2\sqrt{17}$, the larger. Choice C.

24. Trapezoid = $\frac{1}{2}(4+6)5=25$. Semicircle= $\frac{1}{2}\pi(9)=4.5\pi$. 2A= $50+9\pi$ so B+C=59. Choice C.

25. EC= $6\sqrt{3}-5$ so $CD=(6\sqrt{3}-5)\sqrt{2}$ which equals $6\sqrt{6}-5\sqrt{2}$, choice B.

26. The equation of the parabola is $y=-a(x+1)(x-5)$. If the y-intercept is 10 (let x=0) we get a=2. Set x=8, we get y= -54. which is choice B.

27. Volume = $\pi r^2 h-\frac{4}{3}\pi r^3$. Setting h=2r, and simplifying gives $V=\frac{2}{3}\pi r^3$. Setting this equal to 18π gives r=3, and the area of the great circle is $\pi r^2=9\pi$. Choice C.

28. $0.20x+4=0.25(x+4)$ so x=60. Choice C.

29. $\frac{\frac{3}{4}m}{360}(\pi x^2)=\frac{m}{360}(\pi R^2)(\frac{1}{8})$. This solves to $6x^2=R^2$ and so $R=\sqrt{6}x$. Choice C.

30. One of the angles must be 90 degrees. If 9x+9=90 and then x=9. If 4x+4=90 then x=21.5. If kx=90 then the sum of the other two angles is 90 and then 13x+13=90 gives x=77/13. Either way, the sum of all angles is 180 and 13x + 13 + kx = 180. For x=9, k=5.55 and for x=21.5, k is negative. We discard this answer. If x=77/13 then k= 1170/77 which is the larger answer for k, which is choice C.

ANSWERS:

1. A
2. B
3. B
4. A
5. A
6. C
7. A
8. D
9. E
10. A
11. D
12. B
13. C
14. B
15. D
16. B
17. B
18. D
19. A
20. B
21. C
22. D
23. C
24. C
25. B
26. B
27. C
28. C
29. C
30. C