$10^3 - \frac{4\left(\frac{3}{2}\right)^3}{3}\pi = 467.4$

1. C

$x\sqrt{3} = longest\ diagonal$

$\left(3\sqrt{3}\right)^2 = 27$

2. B

$y^2\pi = 225\pi$

$y = 15$

$x = \sqrt{17^2 - y^2} = 8$

3. A

V = $\frac{m}{d} = \frac{357}{3} = 119$

diff height = $\frac{V}{SA\ of\ base} = \frac{119}{50} = 2.38$

4. D

V = $\frac{\pi h}{3}(R^2 + Rr + r^2)$

= $\frac{\pi 36}{3}(8^2 + 8*4 + 4^2)$

$= 1344\pi$

5. A

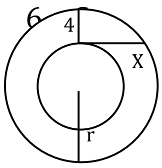
3 faces of $16^2 = 256 * 3$

3 faces of $16^2 - \frac{64}{2} = 224 * 3$

$1\ eq.triangle = 32\sqrt{3}$

= $1440 + 32\sqrt{3}$

6.



$x^2\pi = area\ of\ the\ annulus$

$x = 4\sqrt{10}$

$x^2 = 4(2r - 4)$

$160 = 4(2r - 4)$

$r = 22$

7. A -> Definition

8. C

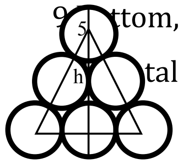
9. D

$V = \frac{4}{3}(12)^3\pi = \frac{(6)^2}{3}\pi$

$4 * 12^3 = 36h$

$h = 64$

10. C



9 bottom, 4 middle, 1 top

tal

$h = 2r + x\ \ and\ x = 2r\sqrt{3}$

$h = 2r + 2r\sqrt{3}$

$10 = 2r(1+\sqrt{3})$

$r = \frac{2}{1 + \sqrt{3}}$

12. D

$5 * 5 * 6 = 150$

13. B

$\frac{bh}{3} = V = \frac{3s^2\sqrt{3}}{2} - \frac{h}{3}$

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

$\sqrt{(36)^2 + (77)^2 + (85)^2} = 85\sqrt{2}$

15. C

$$\frac{4\pi r^2}{3} = 288$$

16. C

$$\frac{4}{3}\pi r^3 = 288\pi$$

$$288\pi = 0.06^2\pi\text{h}$$

h = 80000cm -> 800m

17. A

$$h = \sqrt{\left(\frac{m}{2}\right)^2 + (2m)^2}$$

$$h = 0.5 * \sqrt{17m^2}$$

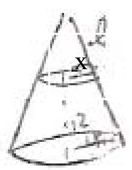
$$l = \sqrt{m^2 + \frac{m^2}{4} + 4m}$$

$$l = 18m^2 = 3m\sqrt{2}$$

18. E

$$n = \frac{2 \times 360}{3 + 37} = 18$$

19. A



$$\frac{15}{12 + x} = \frac{3}{x}$$

$$x = 3$$

$$l = \sqrt{3^2 + 3^2} = 3\sqrt{2}$$

20. B

$$Area = \pi(R + r)\sqrt{(R - r)^2 + h^2}$$

$$= (15 + 3)\pi\sqrt{(15 - 3)^2 + 12^2}$$

$$= 216\pi\sqrt{2}$$

$$V = \frac{\pi}{3}h(R^2 + Rr + r^2)$$

$$= \frac{\pi}{3}12(15^2 + 45 + 3^2)$$

$$= 1116\,\pi$$

$$Sum = \pi(1116 + 216\sqrt{2})$$

21. D

$$A^3 = B^3 = \frac{125}{27}$$

22. B

$$\frac{24}{42 + y} = \frac{18}{y}$$

$$y = \frac{189}{4}$$

$$\frac{x}{18 + \frac{189}{4}} = \frac{18}{\frac{189}{4}}$$

$$x = \frac{144}{7}$$

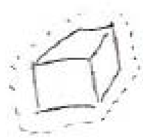
23. D

$$b = 10^2$$

$$lateral\ SA = 4 * 13 * \frac{5}{2} = 260$$

100+260=360

24. A



6 faces = 6*3*36=648

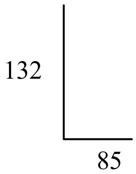
12 edges which are quarter cylinders = $12 * \frac{1}{4} * 9\pi * 6 = 162\pi$

8 corners(1/8 of a sphere) = 8 ∗

$\frac{1}{8} * \frac{4}{3} * 3^3\pi = 36\,\pi$

$648 + 162\,\pi + 36\pi = 648 + 198\pi$

25. A



$\sqrt{132^2 + 85^2} = 157$

26. E

$$V = \frac{\sqrt{2}}{12}s^3 = \frac{\sqrt{2}}{12}4^3 = 16\frac{\sqrt{2}}{3}$$

27. B

$5\sqrt{3}a^2 \rightarrow 5\sqrt{3} * 12^2 = 720\sqrt{3}$

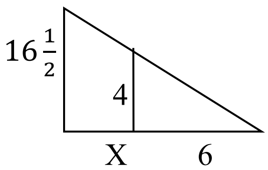
28. A

$6\ by\ 7\ by\ 4\ yards$

$(6 * 7) + 2(7 * 4) + 2(6 * 4)$

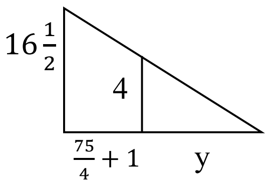
$= 146$

~~29. C~~



$$\frac{6}{4} = \frac{x + 6}{\frac{33}{2}}$$

$x = 75/4$



$$\frac{y}{4} = \frac{\frac{79}{4} + y}{\frac{33}{2}}$$

$y = \frac{158}{25} = 6.32$

~~30. D~~

~~s = 6 yards = 216 inches~~

$\cancel{\frac{216^3}{29} = 347506}$

Mu Alpha Theta National Convention: San Diego, 2013
Theta 3D Geometry Test - Updated Solutions

8 **(C)**. Use coordinate geometry. Let $A = (0, 0, 0)$ and $B = (1, 1, 1)$. If r is the length of the radius of the sphere, then (r, r, r) is its center (in order to be tangent to the three faces that meet at A) and $(r, 1, 1)$ is the point of tangency of one of the edges at B. The distance from (r, r, r) to $(r, 1, 1)$ is equal to the radius, so this means that $r^2 = 2(1 - r)^2$, which has solutions of $r = 2 \pm \sqrt{2}$. We want a point inside the cube, meaning that $r < \sqrt{3}$, so $r = 2 - \sqrt{2}$.

18 **(E)**. The sum of the angle measures of the central angles of the sectors need to equal 360 degrees. If there are n sectors and $a_1 = 3$ and $a_n = 37$ are in arithmetic progression, then

$$360 = \frac{n}{2}(3 + 37),$$

or $n = 18$.

29 **(C)**. Let $A = (4, 0, 0)$, $B = (0, 3, 0)$, $C = (0, 0, 1)$, and $D = (0, 0, 0)$. The formula that relates the volume of the tetrahedron to the radius of the inscribed sphere and the surface area of the tetrahedron is $V = rS/3$. This is analogous to the $A = rp/2$ formula in two dimensions. Taking triangle ADB to be the base of the tetrahedron, $V = (1/3)(1/2 \times 4 \times 3)(1) = (1/3)(6)(1) = 2$. The surface area of the tetrahedron is obtained by adding up the areas of the triangular faces. Since the tetrahedron is composed of trilinear vectors (starting from the origin), the sum of the squares of the three smaller faces is equal to the sum of the squares of the largest face. We have $[ACD] = (1/2)(4)(1) = 2$, $[CDB] = (1/2)(3)(1) = 1.5$, $[ABD] = (1/2)(4)(3) = 6$, and so $[ABC] = \sqrt{2^2 + 1.5^2 + 6^2} = 6.5$, making way for a surface area of $2 + 1.5 + 6 + 6.5 = 16$. Thus $2 = r(16)/3$, so $r = 3/8$, making the volume of the sphere $(4/3)\pi(3/8)^3 = 9\pi/128$. Thus, $m + n = 9 + 128 = 137$.

30 **(D)**. The liquid, when the cone is pointing down with the base horizontal, has half the water level of the big cone, so the liquid is one-eighth the volume of the cone. Thus, when the cone is oriented upwards with the base horizontal, the unoccupied portion of the cone has 7/8ths the volume, which means its height is $\sqrt[3]{7/8}$ths the height of the cone, or $4 \times \sqrt[3]{7/8} = 2\sqrt[3]{7}$.