Theta Equations & Inequalities 2013 MAΘ National Convention

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. Solve: $10x + 9 < 7x + 12$

   (A) $x > 1$ (B) $x < 1$ (C) $x > 3$ (D) $x < 3$ (E) NOTA

2. What conic section is represented by the equation $x^2 - 7 = y^2$?

   (A) circle (B) parabola (C) ellipse (D) hyperbola (E) NOTA

3. What is the slope of the line represented by $3y - 7x + 10 = 0$?

   (A) $\frac{-7}{3}$ (B) $\frac{-3}{7}$ (C) $\frac{10}{3}$ (D) $\frac{3}{10}$ (E) NOTA

4. Suzy is selling *x* hamburgers and *y* hotdogs at the baseball game to raise money for her math team. She's selling hamburgers for \$3 each and hotdogs for \$2 each. She needs to sell at least a total of \$500 worth of a combination of hotdogs and hamburgers to raise the money. Write an inequality to represent how many of each she needs to sell.

   (A) $3x + 2y \geq 500$ (B) $3x + 2y \leq 500$ (C) $x + y > 500$ (D) $x + y < 500$ (E) NOTA

5. Solve the following system, and express the solution as an ordered pair (x, y):
   $2x - 3y = 17$
   $3x + 2y = 6$

   (A) $(3, -4)$ (B) $(3, 4)$ (C) $(4, -3)$ (D) $(4, 3)$ (E) NOTA

6. What points are shared by the line whose equation is $2x - y = 2$ and the circle whose equation is $x^2 + y^2 = 2$?

   (A) $\left(\frac{4 \pm \sqrt{6}}{5}, \frac{-2 \pm 2\sqrt{6}}{5}\right)$ (B) $\left(\frac{4 + \sqrt{6}}{5}, \frac{-2 + \sqrt{6}}{5}\right), \left(\frac{4 - \sqrt{6}}{5}, \frac{-2 - \sqrt{6}}{5}\right)$

   (C) $\left(8 \pm \sqrt{6}, \frac{\pm\sqrt{6}}{5}\right)$ (D) $\left(8 + \sqrt{6}, \frac{\sqrt{6}}{5}\right), \left(8 - \sqrt{6}, \frac{-\sqrt{6}}{5}\right)$ (E) NOTA

7. The line represented by $2(x-7)+y=y-3$ has a slope of

(A) Undefined slope. (B) 0 (C) -2 (D) 2 (E) NOTA

8. The velocity (in feet per second) of an object fired directly upward is given by $V=80-32t$, where t is in seconds. At what time interval will the velocity be between 32 and 64 feet per second (i.e., outside of this interval, the velocity is *not* between 32 and 64 feet per second)?

(A) $1.5<t<3$ (B) $1.5<t<.5$ (C) $1<t<3$ (D) $.5<t<1.5$ (E) NOTA

9. Which of these points is above the graph of $x^2+y=2$?

(A) (2,-2) (B) (4,-5) (C) (0,0) (D) (0,-1) (E) NOTA

10. What point(s), in 3 dimensions, are shared between $x-y=4$ and $2x+3y-z=0$?

(A) $(12,-8)$ (B) $\left(\frac{12}{5},\frac{-8}{5}\right)$ (C) $(12,-8,0)$ (D) $\left(\frac{12}{5},\frac{-8}{5},0\right)$ (E) NOTA

11. Write an equation/inequality to represent the region bounded between the *x*-axis and the line $y=x$.

(A) $0<y<x$ (B) $0<x<y$ (C) $0<y<x$ or $x<y<0$ (D) $0<x<y$ or $y<x<0$ (E) NOTA

12. What is the equation of the translation of the graph of the equation $y=\sqrt{x+1}$ up 7 units and 5 units to the left?

(A) $y=\sqrt{x+7}-5$ (B) $y=\sqrt{x-4}+7$ (C) $y=\sqrt{x+5}+7$ (D) $y=\sqrt{x+6}+7$ (E) NOTA

13. The equation of the line parallel to $2y-x=x+10$ and containing the point (3,10) is

(A) $x-y+7=0$ (B) $x-y-7=0$ (C) $x=0$ (D) $y=7$ (E) NOTA

14. The slanted roof of a store makes a $30°$ angle with the horizontal ceiling. Let the intersection of the roof and ceiling represent the origin and the roof be in the first quadrant. The equation of the line representing the roof is

(A) $y=\frac{1}{2}x$ (B) $y=\sqrt{3}x$ (C) $y=2x$ (D) $y=\frac{\sqrt{3}}{3}x$ (E) NOTA

15. What is the equation of the circle tangent to the *y*-axis whose center is 2 units below the *x*-axis and is on the line $y=x-1$?

(A) $(x-2)^2+(y-1)^2=4$ (B) $(x-1)^2+(y-2)^2=4$
(C) $(x+1)^2+(y+2)^2=1$ (D) $(x+2)^2+(y+1)^2=1$ (E) NOTA

16. Which point is contained in the graph of all 3 inequalities?

$$y\le 2x+3$$
$$x+y<2$$
$$4y>\frac{3}{4}x-\frac{3}{4}$$

(A) (3,1) (B) (-1,1) (C) (1,0) (D) (2,0) (E) NOTA

17. What is the equation for the yz trace of $2x-7y+z=2$?

(A) $2x-7y=2$ (B) $2x+z=2$ (C) $z+7y=2$ (D) $y=\frac{1}{7}z-\frac{2}{7}$ (E) NOTA

18. The dispute box is an open topped rectangular solid with a base x inches by y inches and a height of .5 feet. What is the length of the space diagonal (top corner to opposite bottom corner) of the box, in inches?

(A) $\sqrt{x^2+y^2+36}$ (B) x^2+y^2+36 (C) $\sqrt{x^2+y^2+.25}$ (D) $x^2+y^2+.25$ (E) NOTA

19. Solve: $3(2x-2)+4\ge 2(2x+2)$

(A) $x\le -3$ (B) $-3\le x$ (C) $3\le x$ (D) $x\le 3$ (E) NOTA

20. The school cafeteria is making 500 pancakes for breakfast each day. Each pancake is a perfect circle x inches across at its widest and ¼ inch thick. Which equation represents the amount of pancake batter needed for a Monday through Friday school week?

(A) $625x^2\pi$ in^3 (B) $\frac{625x^2\pi}{4}$ in^3 (C) $375x^2\pi$ in^3 (D) $\frac{375x^2\pi}{4}$ in^3 (E) NOTA

21. Thom Morris is planning to ride the Santa Monica Ferris Wheel. Each "ride" consists of six rotations of the wheel. If the seats are *x* feet from the center of the wheel then how far will he travel in inches during one ride? Express your answer in feet.

(A) $72x\pi$ (B) $144x\pi$ (C) $12x\pi$ (D) $2x\pi$ (E) NOTA

22. Find the geometric mean of all real numbers *x* that satisfy $-2x^5 + 6x^4 + 124x^3 - 692x^2 + 1128x = 864$.

(A) $-6\sqrt[3]{2}$ (B) $-8\sqrt[3]{2}$ (C) -6 (D) 8 (E) NOTA

23. A Norman window has the shape of a rectangle surmounted (placed on top) by a semicircle. If the rectangle's top measure is y and side measure is x, what equation represents the total area of the window?

(A) $A = xy + \frac{y^2\pi}{4}$ (B) $A = xy + \frac{y^2\pi}{8}$ (C) $A = xy + .5y\pi$ (D) $A = xy + y\pi$ (E) NOTA

24. One hundred fifty name pouches were returned from last year's Mu Alpha Theta convention to be recycled for this year. At least 8% of the pouches had been exposed to so much dirt and grime that they were deemed unusable. If 602 name pouches are needed for this convention, which of the following represents how many pouches (p) needs to be ordered?

(A) $p \geq 464$ (B) $p \leq 602$ (C) $p = 464$ (D) $464 \leq p \leq 602$ (E) NOTA

25. Each lane of an eight-lane track is 36 inches wide. The infield of the track is a rectangle with a semicircle curve at each end. If the outermost edge of the infield of the track has a straight side of x yards and the innermost edge of the innermost lane has a diameter of y yards at the curve what expression represents the length of the inside edge of the next to shortest lane? Express your answer in yards.

(A) $2(x-1)+\pi y$ (B) $2x+\pi(y+1)$ (C) $2x+\pi(y+2)$ (D) $2x+\pi y$ (E) NOTA

26. What equation best represents the track of the nose of a wooden horse on a merry-go-round if the horse moves vertically a total of 2 feet during a 5 second interval? Note: the horse is traveling at a speed of 2π feet per 5 seconds.

(A) $y = \sin x$ (B) $y + [x]$ (C) $y = \frac{1}{x}$ (D) $y = \sqrt{x}$ (E) NOTA

27. How many ordered pairs of integers (m, n) are solutions to

$$n^2 + 4n + 2 = (m+1)(m+2)(m+3)(m+4)?$$

(A) 7 (B) 8 (C) 9 (D) 10 (E) NOTA

28. Some surfboard designs are the shape of the intersection of two overlapping congruent circles. If each circle has a diameter of $2r$ feet and their common chord has a length of r feet, which equation represents the area of the surfboard, in square feet?

(A) $A = 2\pi r^2 - \frac{r^2\sqrt{3}}{2}$ (B) $A = \frac{\pi r^2}{3} - \frac{r^2\sqrt{3}}{2}$ (C) $A = \frac{\pi r^2}{6} - \frac{r^2\sqrt{3}}{4}$

(D) $A = \pi r^2 - r\sqrt{3}$ (E) NOTA

29. Calvin and Anna have just filled up their kiddie pool but before they can get in Baxter the bulldog comes running and jumps in! If the pool is a cylinder with a base area of 9π square feet, then which equation represents Baxter's volume if he displaces the water by x inches?

(A) $9x\pi$ ft^3 (B) $9x\pi$ in^3 (C) $\frac{3x\pi}{4}$ ft^3 (D) $\frac{3x\pi}{4}$ in^3 (E) NOTA

30. An alloy needs to contain between 46% copper and 50% copper. Which inequality represents the pounds (x) of a 60% copper alloy that could be mixed with a 40% copper alloy in order to end up with thirty pounds of an alloy containing an allowable percentage of copper?

(A) $0 < x < 7.5$ (B) $9 < x < 18$ (C) $18 < x < 30$ (D) $9 < x < 15$ (E) NOTA