SOLUTIONS:

$$10x + 9 < 7x + 12$$

1. 3x < 3 x < 1

2.
$$x^2 - 7 = y^2$$
$$x^2 - y^2 = 7$$

This is a hyperbola.

$$3y - 7x + 10 = 0$$

- 3. 3y = 7x 10 Slope $= \frac{7}{3}$ $y = \frac{7}{3}x - \frac{10}{3}$
- 4. Multiply \$3 by the number of hamburgers (x) sold and \$2 by the number of hotdogs (y) and add together for a total greater than or equal to \$500. $3x+2y \ge 500$.
- 5. Multiply the top equation by 2 and the bottom equation by 3 to obtain 4x 6y = 34 and 9x + 6y = 18. Add both equations together to get 13x = 52, or x = 4. Substitute to either equation to obtain y = -3.

6. Solve for y - 2x - y = 2 then TIEBREAKERy $\oplus 2x - 2$

substitute in to the circle equation

$$x^{2} + y^{2} = 2$$

$$x^{2} + (2x - 2)^{2} = 2$$

$$x^{2} + 4x^{2} - 8x + 4 = 2$$

$$5x^{2} - 8x + 2 = 0$$

then use the quadratic formula to solve for x and you get

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(2)}}{2(5)}$$
$$x = \frac{8 \pm \sqrt{24}}{10} = \frac{8 \pm 2\sqrt{6}}{10} = \frac{4 \pm \sqrt{6}}{5}$$

Theta Equations & Inequalities

2013 MAO National Convention

Substitute these *x*-values in the

line equation to get the

corresponding y-values.

$$y = 2x - 2$$

$$y = 2\left(\frac{4 + \sqrt{6}}{5}\right) - 2$$

$$y = \frac{8 + 2\sqrt{6}}{5} - 2$$

$$y = \frac{8 + 2\sqrt{6} - 10}{5} = \frac{-2 + 2\sqrt{6}}{5}$$

$$y = 2\left(\frac{4 - \sqrt{6}}{5}\right) - 2$$

$$y = \frac{8 - 2\sqrt{6}}{5} - 2$$

$$y = \frac{8 - 2\sqrt{6} - 10}{5} = \frac{-2 - 2\sqrt{6}}{5}$$

Thus the points are

$$\left(\frac{4+2\sqrt{6}}{5}, \frac{-2+4\sqrt{6}}{5}\right)$$

and

$$\left(\frac{4+2\sqrt{6}}{5}, \frac{-2-4\sqrt{6}}{5}\right).$$

$$2(x-7)+y=y-3$$
$$2x-14+y=y-3$$
7. $2x = 11$

$$x = \frac{11}{2}$$

This line has no slope.

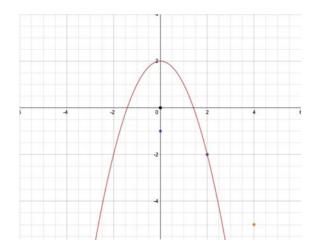
$$32 < 80 - 32t < 64$$

 $32 < 80 - 32t$
 $32t < 48$
 $t < 1.5$

8.
$$80 - 32t < 64$$

 $16 < 32t$
 $.5 < t$

9. Looking at the graph



You'll see that (4,-5) is the only point above the graph.

$$x-y=4$$

$$x = 4 + y$$

$$z = 0$$

$$2x+3y-z=0$$

$$2(4+y)+3y-0=0$$

$$10. 8+2y+3y=0$$

$$8+5y=0$$

$$5y=-8$$

$$y = \frac{-8}{5}$$

$$x = 4 + \frac{-8}{5} = \frac{12}{5}$$

$$\left(\frac{12}{5}, \frac{-8}{5}, 0\right)$$

11. If you look at the graph, then the region in the first quadrant is represented by 0 < x < y and the region in the third quadrant is represented by y < x < 0.

12.
$$y = \sqrt{(x+5)+1} + 7$$
$$y = \sqrt{x+6} + 7$$

13. Find the slope of the original line

$$2y-x = x+10$$
$$2y = 2x+10$$
$$y = x+5$$
$$m = 1$$

Then find the equation of the line through (3,10) with slope m = 1

$$y-10 = 1(x-3)$$

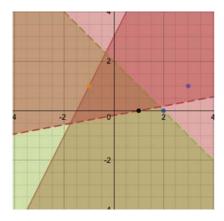
 $y-10 = x-3$
 $y = x+7$

angle, the slope of the line will be the ratio of the legs of a 30/60/90 right triangle. The leg opposite the 30° angle is the rise and the other the run. Thus the slope is

$$m = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
 and the equation for
the line would be $y = \frac{\sqrt{3}}{3}x$.

15. Since the center is 2 units below the x-axis then the y-value is -2. Plug that in to the line equation to y = x - 1 find x. -2 = x - 1 so the center of -1 = x the circle is at (-1,-2). Since the circle is tangent to the y-axis, the radius of the circle is 1 and the equation is $(x+1)^2 + (y+2)^2 = 1$

16. From the graph



you can see that (-1,1) is the only point contained in all 3 graphs.

17. To find the yz trace, set x = 0.

$$2(0)-7y+z=2$$

$$-7y+z=2$$
Then you get $-7y=2-z$

$$y=\frac{1}{7}z-\frac{2}{7}$$

18. To find the diagonal, you first find the diagonal of the base

 $D = \sqrt{x^2 + y^2}$. This diagonal and the height of the box create a right triangle where the diagonal of the box is the hypotenuse. We convert the height to inches (6) then we see: The diagonal

$$= \sqrt{\left(\sqrt{x^2 + y^2}\right)^2 + 6^2}$$
$$= \sqrt{x^2 + y^2 + 36}$$

$$3(2x-2)+4 \ge 2(2x+2)$$
19.
$$6x-6+4 \ge 4x+4$$

$$2x \ge 6$$

$$x \ge 3$$

20. Formula is found by multiplying days by number of pancakes by volume of each pancake.

$$5(500)(\pi(\frac{x}{2})^{2}(\frac{1}{4}))$$

$$= 2500(\frac{\pi x^{2}}{16})$$

$$= \frac{625\pi x^{2}}{4}$$

- 21. 6 rotations would cover the circumference 6 times. $6(2\pi x)=12\pi x \text{ ft and then we}$ convert feet to inches to see he goes $12(12\pi x)=144x\pi$ inches.
- 22. The polynomial $-2x^5 + 6x^4 + 124x^3 692x^2 + 1128x = 864$ has real roots of 4, 6, and -9, so the geometric mean is the cube root of -216, or -6.
- 23. Area of the rectangle = xy. And the area of the semi-circle $= \left(\frac{1}{2}\right)\pi \left(\frac{y}{2}\right)^2 = \frac{y^2\pi}{8} \text{ so the total}$

area is the sum of those

$$A = xy + \frac{y^2 \pi}{8}.$$

The upper limit on pouch ordering is 602, hence D.

- 25. To find the length of the inside edge of the next to shortest lane we add the straight sides and get 2x. And then we need to add the circumference of the semi-circular ends. The width of the lane (36in=1yd) adds 2 yards to the diameter so you get a circumference of $(y+2)\pi$. The equation for the length of the inside edge of the lane is $xy + \pi(y+2)$.
- 26. Imagine the nose as it moves forward as well as up and down. It creates the image of a Sin wave and given the information, the equation would be $y = \sin x$.

- 27. The left side of the equation either leaves are remainder of 2 or 3 upon division by 4. The right hand side, however, being a product of 4 consecutive integers, is always divisible by 4! = 24; hence always divisible by 4. There are no solutions to the Diophantine Equation.
- 28. If you know the chord has a length of r and the diameter is 2r then triangle made by the chord and the radii connecting to each end create an equilateral triangle of side r. By finding the area of the sector ($\frac{1}{6}$ of the whole circle) and subtracting the area of the triangle, you will get $\frac{1}{2}$ the area of the surfboard.

$$A = 2\left(\frac{2\pi r^2}{6} - \frac{r^2\sqrt{3}}{4}\right)$$
$$A = \frac{2\pi r^2}{3} - \frac{r^2\sqrt{3}}{2}$$

29. Since Baxter changed the height of the water by x inches ($\frac{x}{12}$ feet) then his volume can be found by

$$V = 9\pi \left(\frac{x}{12}\right)$$
$$V = \frac{3\pi x}{4}$$

30. Total is 30lbs so if there are x lbs of the 60% alloy then there are 30-x lbs of the 40% alloy. Thus we need

$$.46 < \frac{.60x + .40(30 - x)}{30} < .5$$

$$.46 < \frac{.60x + .40(30 - x)}{30}$$

$$13.8 < .6x + 12 - .4x$$

$$\frac{.60x + .40(30 - x)}{30} < .5$$

$$.6x + 12 - .4x < 15$$
 then
$$.2x < 3$$

x < 15

TIEBREAKER:

Swap g and f and solve for g.

$$f = 12g - 4$$

$$f + 4 = 12g$$

$$g^{-1}(f) = \frac{f+4}{12}$$