Solutions – Equations & Inequalities
Theta Page 1

1. $ax + by = c$

$\frac{-b}{a} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{5-1} = \frac{4}{4} = 1$

$a = 1 \quad b = -1$

$x - y = c$

Plug in $(1, 3)$, $c = -2$

$x - y = -2$ C

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2. $3(3x-3) - (3x-1) = 8 + 3(3-3x)$

$9x - 9 - 3x + 1 = 8 + 9 - 9x$

$15x = 25 \quad x = 5/3$ B

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3. ~~$x^2 - x - 12 = 0$~~ OK

~~$(x-4)(x+3) = 0$~~

~~roots are 4, -3~~ D "roots" is proper term

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4. $p(x) = -100x^2 + 700x + 15000$

The maximum of a parabola occurs at $\frac{-b}{2a}$, $p(x)$ maximized at $\frac{-700}{2(-100)} = \frac{7}{2}$ D

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5. $\ln x + \ln(2x) = 2$

$e^{(\ln x + \ln(2x))} = e^2$

$x(2x) = e^2 \quad x^2 = \frac{e^2}{2} \quad x = e\frac{\sqrt{2}}{2}$ B

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6. $a+b = 10$
$b+c = -7$
$c+d = 3$
$d+e = 11$
$+ \quad e+a = 1$

$a+b+c+d+e = 9$

$2a = -4$ $10 - 17 + 3 = 16$, 1

Solutions – Equations & Inequalities
Theta Page 2

7. $3^x = 8$ $\qquad 3^{2x} = 64 \qquad 9^x = 3^{2x}$

~~$x \ln 3 = \ln 8$~~ $\quad 9^x - 5 \cdot 3^{2x} + 4 = \quad 64 - 5(64) + 4 = -252$ C

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8. $\sqrt{x^2 - 4x - 38} = \sqrt{7}$

$x^2 - 4x - 38 = 7$

$x^2 - 4x - 45 = 0$ → $(x-9)(x+5) = 0$

$x = -5$ or 9

C

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9. $\sqrt{5 - \sqrt{7+q}} = 1$

$-\sqrt{7+q} = -4$

$7 + q = 16 \qquad q = 9$ B

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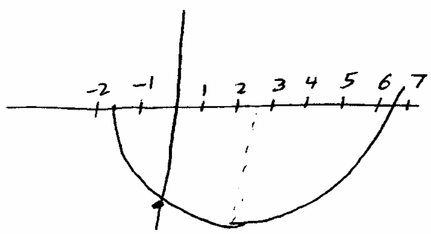
10. $x^2 - 5x - 11 \le 0$ $\qquad \frac{-b}{2a} = \frac{5}{2}$

Graphical Technique



The set $\{-1, 0, 1, \ldots, 6\}$ satisfies the condition A

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11. Bart $\ge$ Arthur $+ \frac{200}{100}$ Arthur

Bart $\ge 3 \cdot$ Arthur

Carl $\ge$ Bart $+ \frac{400}{100}$ Bart $= 5 \cdot$ Bart

Carl $\ge 15 \cdot$ Arthur

Dan $\ge$ Carl $+ \frac{40}{100}$ Carl $= 1.4$ Carl

Dan $\ge 21 \cdot$ Arthur $\qquad$ Arthur $\le \frac{\text{Dan}}{21} = \$200{,}000$ A

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12. $x^2 + 5x + 4 > 0$

$(x+4)(x+1) > 0$

case $x = -5$: $x^2 + 5x + 4 = 4$

case $x = -2$: $x^2 + 5x + 4 = -2$

case $x = 0$: $x^2 + 5x + 4 = 4$

$x > -1$ or $x < -4$ D

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13. $x^3 = -8$

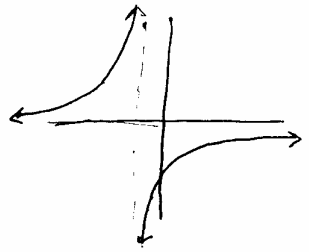
$x =$ [illegible] $120°$, [illegible] $(-120°$

$x = -2, -2(\cos 120 + i \sin 120), -2(\cos -120 + i \sin -120)$ D

Solutions – Equations & Inequalities
Theta page 3

14. $f(x)=\frac{\sqrt{4-x}}{x-3}$ $x\neq 3,\ x\leq 4$ D

15. $\frac{x-3}{2x+1} > 4$



$x=-\frac{1}{2}$: special case

$x=0$: $\frac{x-3}{2x+1} = -3$

$x=-1$: $\frac{x-3}{2x+1} = 3$ A

16. $x=\frac{6}{6-3y}$

$6x-3xy=6$

$y=\frac{6-6x}{-3x} = 2-\frac{2}{x}$ ~~$\frac{2}{x}$~~ $= \frac{2x-2}{x}$ A

17. $f(14)=\frac{14^2}{9}=\frac{196}{9}$ $9\left(\frac{196}{9}\right)=9\left(\frac{196}{9}\right)+4=200$ D

18. $2^{f-1}32^{f+1}=8^{3f-3}$

$2^{f-1}2^{5f+5}=2^{9f-9}$

$6f+4=9f-9$

$f=\frac{13}{3}$ D

19. $6^{x-2}18^{y+2}$

$2^{x-2}3^{x-2}2^{y+2}3^{2y+4} = 2^6\cdot 3^{10}$

$x-2+y+2=6$ $x-2+2y+4=10$

$y=-x+6$ $y=\frac{-x}{2}+4$

$x=4$ $y=2$ C

20. $x^3-2x^2+4x-8=0$ $x=2$

$x-2\ \overline{)\ x^3-2x^2+4x-8}$ $\quad x^2+4$

$-x^3+2x^2$

$4x-8$

$-4x+8$

0

$x=2, 2i, -2i$ $2i-2i=0$

A

21. $\frac{-D}{E} = \frac{-4}{3}$

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22. $f(x-3) = x^2 + 6x - 11$

Let $u = x-3$

$u^2 = x^2 - 6x + 9$

$f(x-3) = u^2 + 12x - 20 = f(u)$

$f(u) = u^2 + 12u + 16$

$\frac{f(x)}{5} = x^2 + 12x + \frac{16}{5}$ B

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23. $\frac{4x}{x^2-4x-5} = \frac{A}{x+1} + \frac{B}{x-5}$

$4x = A(x-5) + B(x+1)$ $A+B=4$ C

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24. $\frac{32x-9}{x^2+2x+1} = \frac{A}{x+1} + \frac{B}{(x+1)^2}$

$32x-9 = A(x+1) + B$ $A+B=-9$ C

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25. $x+y+z=1$
$3x-2y+3z=5$

$3x-2y+3z=5$
$2x+2y+2z=2$

$5x = 7-5z$

$x = \frac{7}{5} - z = -z + \frac{7}{5}$ A

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26. $-3x+4y+2z=-15$
$3x-4y-4z=13$

$-2z=-2$

$z=+1$

$16x-4y-16z=40$
$-3x+4y+2z=-15$

$13x - 14 = 25$

$x=3$ $y=-2$

$x+y+z=2$ C

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27. $\log_2(5x) + \log_8(8x^3) = 1$

$\log_2(5x) + \frac{1}{3}\log_2(8x^3) = 1$

$\log_2(5x)((8x^3)^{\frac{1}{3}}) = 1$

$\log_2(5x)(2x) = 1$

$10x^2 = 2$

$x = \frac{\sqrt{5}}{5}$ B

28. $\log_3(x) = \log_x(27)$

$$\frac{\log_{10}(x)}{\log_{10}(3)} = \frac{\log_{10}(27)}{\log_{10}(x)}$$

$$[\log_{10}(x)]^2 = \log_{10}(3)\log_{10}(27)$$

$$\log_{10}(x) = \pm\sqrt{\log_{10}(3)\log_{10}(27)}$$

$$x = 10^{\pm\sqrt{\log_{10}(3)\log_{10}(27)}}$$ C

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29. I want $\lfloor \log_2 N \rfloor$ and $\log_2 N$ the greatest possible distance apart. This is when $N = 2^x - 1$ where x is any positive integer.
In this problem, x is constrained to be 9 or less.
The lowest value is $8 - \log_2 511$ C

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30. $\frac{x-2}{-4x+5} \leq -3$

$x - 2 \leq 12x - 15$, $x \neq \frac{5}{4}$

$13 \leq 11x$

$\frac{13}{11} \leq x$, $x < \frac{5}{4}$ D

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31. Bill ~~+5~~ = Mary + 9

Bill + 5 = 2(Mary − 3)

− Bill = − Mary − 9

5 = Mary − 15

Mary = 20 Bill = 29 D

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32. $r = -3$ $3r - qr^3 + 11 = 5$

$-9 + 27q + 11 = 5$

$q = \frac{1}{9}$ C

33. $3x^2 + 18x + c = 0$

case I: $\frac{-18 + \sqrt{324-12c}}{6} = 2\left(\frac{-18 - \sqrt{324-12c}}{6}\right)$

$18 + \sqrt{324-12c} = -2\sqrt{324-12c}$

This is impossible.

case II: $2(-18 + \sqrt{324-12c} = -18 - \sqrt{324-12c}$

$3\sqrt{324-12c} = 18$

$324-12c = 36 \quad c = 24$ D

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34. $A + B = 2$
$D + E = 3$
$A + D = 2$
$A + E = 3$

$D + E = 3$
$-D - A = -2$
$E - A = 1$
$+ E + A = 3$

$E = 2, \ A = 1 \quad D = 1 \quad B = 1$ A

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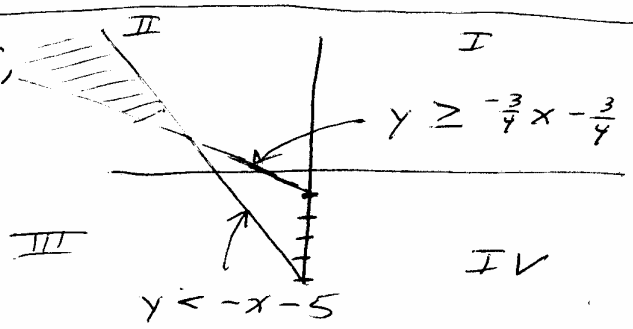
35. $m = 7k^{-1}(m) - 3$

$k^{-1}(m) = \frac{m+3}{7}$ B

36.



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37. D hyperbola

38. $f(7) = \frac{7}{5}$

$g(\frac{7}{5}) = 2(\frac{7}{5}) - 7 = \frac{14}{5} - \frac{35}{5} = \frac{-21}{5}$ B

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39. $3w - 44 < 35$

$3w < 79$

$w < \frac{79}{3}$ D

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40. $t - 1 < -t + 4$ $\quad -t + 4 < 2t + 17$

$2t < 5$ $\quad -13 < 3t$

$t < \frac{5}{2}$ $\quad \frac{-13}{3} < t$

$t = \{2, 1, 0, -1, -2, -3, -4\}$ D