Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. The sum of five positive integers is what percent of the arithmetic mean of the same five integers?

    (A) 5%   (B) 50%   (C) 500%   (D) 5000%   (E) NOTA

2. If the standard form of a quadratic equation is $ax^2 + bx + c = 0$ where $a > 0$ and a, b, and c are integers with no common factors, find $a + b + c$ in the quadratic equation with the solution set $x \in \{1+3i, 1\text{-}3i\}$. Note, here, $i = \sqrt{-1}$.

    (A) 8   (B) -2   (C) 9   (D) 13   (E) NOTA

3. The outside dimensions of a picture frame are 36 inches by 48 inches. The frame has a uniform width. What is the frame width if the inside dimensions are in the ratio 5 to 7?

    (A) 3 inches   (B) 6 inches   (C) $7\frac{1}{5}$ inches   (D) $6\frac{6}{7}$ inches   (E) NOTA

4. Simplify $\left(\dfrac{1}{y^{-1} + x^{-1}} \right)(x + y)$ given $x \neq 0, y \neq 0$.

    (A) $x^2 + y^2$ (B) $x^2 + 2xy + y^2$ (C) $\dfrac{(x+y)^2}{xy}$ (D) $\dfrac{x^2 + y^2}{xy}$ (E) NOTA

5. If $f(x) = k(2 - x - x^3)$ is a one-to-one function and $f^{-1}(3) = -2$. Find the value of k.

    (A) $\dfrac{1}{4}$ (B) $\dfrac{1}{56}$ (C) $\dfrac{1}{14}$ (D) $\dfrac{-2}{3}$ (E) NOTA

6. Find the inverse of $f(x) = \dfrac{x-2}{x-1}$.

    (A) $f^{-1}(x) = \dfrac{x-2}{x-1}$ (B) $f^{-1}(x) = \dfrac{x+2}{x+1}$ (C) $f^{-1}(x) = \dfrac{x-1}{x-2}$ (D) $f^{-1}(x) = -\dfrac{x-2}{x-1}$ (E) NOTA

7. The height, at any time t (in seconds), above the ground of an object thrown upward from a point s_0 meters above the ground with an initial velocity of v_0 meters per second is given by $f(t) = -4.9t^2 + v_0 t + s_0$. If a baseball is thrown upward from a height of 2 meters with an initial velocity of 10 meters per second, determine the maximum height of the baseball, correct to 1 decimal place.

   (A) 1.0 meter   (B) 7.1 meters   (C) 8.5 meters   (D) 20.2 meters   (E) NOTA

8. Find the reciprocal of $\dfrac{3}{x+1} + \dfrac{2}{x}$.

   (A) $\dfrac{2x+1}{5}$ (B) $\dfrac{5x+2}{6}$ (C) $\dfrac{x^2+x}{5}$ (D) $\dfrac{x^2+x}{5x+2}$ (E) NOTA

9. After expanding $f(x)$ and all the like terms are combined, how many terms does $f(x) = (x+y)^7 + (x-y)^7$ have?

   (A) 16 terms   (B) 8 terms   (C) 7 terms   (D) 4 terms   (E) NOTA

10. Consider the equation $\dfrac{x+4}{(x+1)(x+2)} = \dfrac{Q}{x+1} + \dfrac{R}{x+2}$ $(x \neq -1, x \neq -2)$. In order to solve for Q and R, which of the following system of equations would be most useful?

   (A) $\begin{cases} Q+R=2 \\ Q-R=1 \end{cases}$ (B) $\begin{cases} x+2=Q \\ x+1=R \end{cases}$ (C) $\begin{cases} Q+R=1 \\ 2Q+R=4 \end{cases}$ (D) $\begin{cases} Q+R=4 \\ 4Q+2R=1 \end{cases}$ (E) NOTA

11. Find an equation of the tangent line to the circle $x^2 + y^2 + 10x - 6y = -16$ at the point (-2,6).

   (A) $y = x+8$ (B) $y = -x+4$ (C) $y = \dfrac{-3x+24}{5}$ (D) $y = \dfrac{3x+36}{5}$ (E) NOTA

12. If n is a positive integer, simplify $\dfrac{n(n+1)! - 2(n!)}{(n+1)! + n!}$ completely.

   (A) $n-2$ (B) n (C) $n-1$ (D) $(n+2)!$ (E) NOTA

13. Using interval notation, state the domain and range of the real-valued function
    $f(x) = \sqrt{9 - x^2}$.

    (A) Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ (B) Domain: $[-3,3]$ Range: $[-3,3]$
    (C) Domain: $[-3,3]$ Range: $[0,3]$ (D) Domain: $(-\infty,-3]\cup[3,\infty)$ Range: $(-\infty,\infty)$ (E) NOTA

14. Which of the following statement is false about functions:
    I. A function is even if $f(x) = f(-x)$ for all x in the domain of f.
    II. A function is odd if $f(x) = -f(-x)$ for all x in the domain of f.
    III. A function can be neither odd nor even.
    IV. A function can be both odd and even.
    V. Any function can be written as the sum of an odd and an even function.

    (A) IV   (B) IV, V   (C) V   (D) III, IV   (E) NOTA

15. If $f(x) = ax^4 + x^3 - cx^2 + 5x + 1$ and $f(-4) = 10$, what is the value of $f(4)$?

    (A) -10   (B) 268   (C)10   (D) 178   (E) NOTA

16. According to Newton's Law of Gravitation, the gravitational force between two planets is
    directly proportional to the product of their masses, and inversely proportional to the
    square of the distance separating them. Consider three planets: Planets *Mu* and *Alpha* have
    equal masses, while Planet *Theta* has twice the mass of Planet *Mu*. For which arrangements
    below will the gravitational force be greatest?

    (A) The force between Planets Mu and Alpha when they are separated by a distance *d*.
    (B) The force between Planets Mu and Theta when they are separated by a distance 2*d*.
    (C) The force between Planets Alpha and Theta when they are separated by a distance *d*.
    (D) The force between Planets Mu and Alpha when they are separated by a distance *d*/2.
    (E) NOTA

17. If $f(x) = \dfrac{1}{1-x}$ and $g(x) = f(f(f(x)))$ what kind of function is g ?

    (A) Piecewise   (B) Quadratic   (C) Linear   (D) Hyperbolic   (E) NOTA

18. If $f(x) = |x-3| + |x-1|$ for all real x, how many solutions does $f(t+2) = f(t)$ have?

    (A) 0   (B) 1   (C) 2   (D) Infinitely many   (E) NOTA

19. The quantities x, y, and z are positive and obey the relation $x^2 y = \dfrac{z}{4y^2}$. If x is increased by

    25% and y is decreased by 20%, how must z be changed so that the relation $x^2 y = \dfrac{z}{4y^2}$

    stays the same?

    (A) z must be increased 20%.
    (B) z must not change.
    (C) z must be decreased by 20%.
    (D) z must be decreased 25%.
    (E) NOTA

20. A theatrical stage is designed as a semicircle of radius R, with the flat edge of the semicircle at the back of the stage. To help actors find their mark, a line is drawn across the stage halfway from the back to front. What fraction of the stage's area is in front of the line?

    (A) $\dfrac{2}{3} - \dfrac{\sqrt{3}}{2\pi}$　　(B) $\dfrac{4\pi - 3\sqrt{3}}{6\pi}$　　(C) $\dfrac{\sqrt{3}R^2}{4}$　　(D) $\left(\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{4}\right)R^2$　(E) NOTA

21. If (3, 12), (-2, -3), and (4, 21) lie on a parabola, what is the value of y when $x = 2$?

    (A) 10　　　　(B) -5　　　　(C) 5　　　　(D) 6　　　　(E) NOTA

22. The manager of an electronics store can sell 21 cell phones at $120.00 each. She estimates that for each $5.00 decrease in price, she will sell three more phones. She buys the phones from a wholesale dealer at a rate of $75.00 each. If she sells as many phones as she buys, write a quadratic function to describe the owner's profit after she decreases the price. Let n = the number of $5 price decreases.

    (A) Profit = $63 - 57n - n^2$　(B) Profit = $945 - 240n^2$　(C) Profit = $945 + 30n - 15n^2$
    (D) Profit = $2520 + 21n - 24n^2$　　(E) NOTA

23. Suppose we have the relations $f = \{(0,-3),(2,5),(-1,1),(4,2)\}$　$g = \{(-1,2),(1,4),(4,3),(0,-1)\}$, and $h = \{(4,2),(1,0),(-3,4),(3,-1)\}$. Find the relation that represents $f \circ g \circ h$.

    (A) $\{(3,5),(1,1)\}$　(B) $\{(3,5),(1,2)\}$　(C) $\{(-1,2)\}$　(D) $\{(4,5),(1,-3),(-3,2)(3,1)\}$　(E) NOTA

24. A scientist has a 30 gallon mixture of water, oil, and ethanol. This mixture is 30% water and includes 2 gallons of oil. She wants to add 5 gallons of oil and an unknown amount of ethanol to this mixture to make a new mixture that is 20% water. How much ethanol must she add?

   (A) 5 gallons   (B) 15 gallons   (C) 29 gallons   (D) 10 gallons   (E) NOTA

25. The Whispering Chamber at the local science museum is in the shape of an ellipse. If two students stand at the foci with their backs to each other, even a slight whisper from one student will be reflected to the other student's ear. The students are located 8 meters apart, each facing a wall one meter away. Find the length of the minor axis of the ellipse.

   (A) 3 meters   (B) 4 meters   (C) 5 meters   (D) 9 meters   (E) NOTA

26. Find the sum of the zeroes of the function $f(a) = 2a^{1/2} - 13a^{1/4} + 20$.

   (A) $\dfrac{4721}{16}$ (B) 1000   (C) $\dfrac{13}{2}$ (D) $\dfrac{28561}{16}$ (E) NOTA

27. How many unique, real ordered pairs (x, y) are solution(s) of $\begin{bmatrix} x & y \\ y & x \end{bmatrix}^2 = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$?

   (A) 0   (B) 2   (C) 4   (D) infinitely many   (E) NOTA

28. Jack is trying to guess Jill's phone number. He knows the first three digits of the exchange, and he knows the last four digits are positive integers that add to get 7. What is the probability that Jack guesses Jill's phone number on the first guess?

   (A) $\dfrac{1}{9^4}$ (B) $\dfrac{1}{20}$ (C) $\dfrac{1}{18}$ (D) $\dfrac{1}{4 \cdot 9^3}$ (E) NOTA

29. Over a distance of 100 kilometers, I drove at a constant speed of 10 kilometers per hour faster than my sister. I arrived at our destination 30 minutes before she did. What was my speed?

   (A) 50 km/hour (B) 40 km/hour (C) 60 km/hour (D) $5\left(\dfrac{1+\sqrt{21}}{3}\right)$ km/hour   (E) NOTA

30. Suppose $f(x)$ is a real-valued function such that $5f\left(\dfrac{1}{x}\right)+\dfrac{f(2x)}{x^2}=x$ for $x\neq 0$. Find $f(1)$.

(A) $\dfrac{3}{10}$ (B) $\dfrac{1}{21}$ (C) $\dfrac{1}{7}$ (D) $\dfrac{3}{14}$ (E) NOTA