Individual Theta Test #231

1. Write your 6-digit ID# in the I.D. NUMBER grid, left-justified, and bubble. Check that each column has only one number darkened.
2. In the EXAM NO. grid, write the 3-digit Test # on this test cover and bubble.
3. In the Name blank, print your name; in the Subject blank, print the name of the test; in the Date blank, print your school name (no abbreviations).
4. Scoring for this test is 5 times the number correct + the number omitted.
5. You may not sit adjacent to anyone from your school.
6. TURN OFF ALL CELL PHONES OR OTHER PORTABLE ELECTRONIC DEVICES NOW.
7. No calculators may be used on this test.
8. Any inappropriate behavior or any form of cheating will lead to a ban of the student and/or school from future national conventions, disqualification of the student and/or school from this convention, at the discretion of the Mu Alpha Theta Governing Council.
9. If a student believes a test item is defective, select "E) NOTA" and file a Dispute Form explaining why.
10. If a problem has multiple correct answers, any of those answers will be counted as correct. Do not select "E) NOTA" in that instance.
11. Unless a question asks for an approximation or a rounded answer, give the exact answer.

Note: For all questions, answer "E) NOTA" means none of the above answers is correct.

1. Of the following terms, all of which describe the same shape, which is the most specific?

A) parallelogram B) quadrilateral C) square D) rectangle E) NOTA

2. If $f(x)=-3x+2$, what is the value of $f\left(f\left(f(-1)\right)\right)$?

A) -1 B) 41 C) -17 D) 53 E) NOTA

3. Consider the function $f(x)=\dfrac{20x^3+31x^2-31x-42}{15x^2+17x+2}$. Which of the following are asymptotes of the graph of $y=f(x)$?

I) $x=-1$ II) $x=-\dfrac{2}{15}$ III) $x=\dfrac{6}{5}$ IV) $y=\dfrac{4}{3}$ V) $y=\dfrac{4}{3}x+\dfrac{5}{9}$

A) I & II only B) II only C) II & IV only D) II & V only E) NOTA

4. Solve the following system of linear equations for z: $\begin{cases} x+2y=3 \\ 4x+5z=6 \\ 7y+8z=9 \end{cases}$

A) $\dfrac{10}{33}$ B) $\dfrac{18}{33}$ C) $\dfrac{31}{33}$ D) $\dfrac{37}{33}$ E) NOTA

5. Three distinct points A, B, and C lie on a circle. Chords $\overline{AC}$ and $\overline{BC}$ have the same length, and minor arc AC has measure equal to three times the measure of $\angle ACB$. Find the measure of major arc ABC.

A) 225° B) 240° C) 255° D) 270° E) NOTA

6. Find the vertex of the parabola with equation $-6x=(y+8)(y-4)$.

A) $(-2,6)$ B) $(0,4)$ C) $(0,-8)$ D) $(10,-18)$ E) NOTA

7. Find the number of distinct linear permutations of the letters in the word HOTELDEL.

A) 5040 B) 10,080 C) 20,160 D) 40,320 E) NOTA

8. The region in the plane described by $(x-3)^2+(y-4)^2 \le 7$ is rotated about the x-axis. Find the volume of the solid generated by this rotation.

A) $28\pi^2$ B) $42\pi^2$ C) $56\pi^2$ D) $70\pi^2$ E) NOTA

9. A regular hexagon is inscribed in an equilateral triangle of side length 16 such that each of the six vertices of the hexagon lie on a side of the equilateral triangle. Find the area enclosed by the hexagon.

A) $32\sqrt{3}$ B) $\frac{128\sqrt{3}}{3}$ C) $48\sqrt{3}$ D) $\frac{160\sqrt{3}}{3}$ E) NOTA

10. A sphere with radius 12 is centered at the point $Q(-2,-3,5)$. The point $P(1,-5,-1)$ is contained within the sphere; what is the shortest distance from P to the surface of the sphere?

A) 5 B) 6 C) 7 D) 8 E) NOTA

11. Find the nonzero value of x that makes the matrix $A=\begin{bmatrix} 2 & x & 1 \\ 1 & -x & 4 \\ x & 0 & -5 \end{bmatrix}$ singular.

A) 1 B) -1 C) -3 D) -5 E) NOTA

12. A triangle has interior angles of measure 45°, 60°, and 75°. If the side opposite the 60° angle has length $4\sqrt{6}$, find the area enclosed by the triangle.

A) $9+16\sqrt{3}$ B) $16+12\sqrt{3}$ C) $6+18\sqrt{3}$ D) $24+8\sqrt{3}$ E) NOTA

13. Two distinct lines L and M are each perpendicular to a third line. Which of the following terms must describe the relationship between L and M?

A) skew B) intersecting C) parallel D) perpendicular E) NOTA

14. A circle has chords $\overline{AB}$ and $\overline{CD}$ that intersect at point E. The lengths of $\overline{AE}$, $\overline{CE}$, and $\overline{BE}$ are 6, 15, and 35, respectively. Find the length of $\overline{DE}$.

A) 10 B) 12 C) 14 D) 16 E) NOTA

15. Bjorn selects one number from each of the three sets $A=\{1,2,6,10\}$, $B=\{2,7,9,11\}$, and $C=\{1,3,5,14\}$, then multiplies those three numbers together. Find the sum of all such possible products Bjorn could obtain in this way. If a product can be formed in more than one way, add the product one time for each time it is formed.

A) 11,799 B) 12,673 C) 19,364 D) 33,600 E) NOTA

16. The arithmetic mean of three test scores is a 92. After taking the fourth test, the arithmetic mean of all test scores is now 94. Find the score of the fourth test.

A) 93 B) 96 C) 98 D) 100 E) NOTA

17. A convex decagon has interior angles with integer degree values. At least nine of the interior angles have the same measure (the last angle may or may not have the same measure). What is the smallest possible degree measure of each of the nine congruent angles?

A) 141° B) 140° C) 139° D) 138° E) NOTA

18. An infinite geometric sequence has third term 45 and fifth term 5. Find the sum of all series of all possible geometric sequences satisfying this condition.

A) $\frac{1215}{4}$ B) $\frac{1215}{2}$ C) $\frac{3645}{4}$ D) $\frac{3645}{2}$ E) NOTA

19. How many lattice points (x,y) satisfy the inequality $(x+2)^2+(y-3)^2\le 25$?

A) 81 B) 87 C) 69 D) 75 E) NOTA

20. A right circular cylinder has base radius 8 and height 5. Marcus shoves solid spherical Play-Doh balls of radius 2 into the cylinder with his goal being to get as many balls into the cylinder as possible. Marcus doesn't care if the balls retain their shape either. In order to reach his goal, how many whole spherical balls could Marcus fit inside the cylinder?

A) 18 B) 22 C) 26 D) 30 E) NOTA

21. Solve for positive real number x: $x^4=x^2+2x+1$

A) $\frac{1+\sqrt{5}}{2}$ B) $\frac{-1+\sqrt{5}}{2}$ C) $\frac{3+\sqrt{5}}{2}$ D) $\frac{3-\sqrt{5}}{2}$ E) NOTA

22. Find the value of $\sum_{n=1}^{\infty}\left((2n+3)\left(\frac{2}{3}\right)^n\right)$.

A) 18 B) 21 C) 24 D) 27 E) NOTA

23. A circle that encloses an area of $\frac{\pi}{4}$ is inscribed in a regular hexagon and has a regular hexagon inscribed in it. Using the perimeters of these hexagons as estimates for π yields what inequality?

A) $3<\pi<3\sqrt{3}$ B) $3<\pi<2\sqrt{3}$ C) $2\sqrt{2}<\pi<3\sqrt{2}$ D) $2\sqrt{2}<\pi<2\sqrt{3}$ E) NOTA

24. For the logical statement "$A\rightarrow(B\vee C)$", knowing which of the following conditions has a truth value of True makes the statement have a truth value of True?

I) $\sim A$ II) $\sim B\rightarrow\ \sim C$ III) $\sim B\rightarrow\ \sim A$

A) I only B) II only C) I & III only D) I, II, & III E) NOTA

25. The sum of the interior angles of a triangle drawn on the surface of a hemisphere has a least upper bound of how many degrees?

A) 540° B) 360° C) 270° D) 180° E) NOTA

26. Five points are placed on a circle with radius 1 such that the distance between any two consecutive points on the circle is the same. From one of these points, the four chords connecting that point to each of the other four points is drawn. What is the product of the lengths of these four chords?

A) 3 B) 4 C) 5 D) 6 E) NOTA

27. Two circles whose radii have different lengths are externally tangent. A common external tangent to the two circles is drawn, and the distance between the points of tangency of each circle with the common external tangent equals the length of the larger circle's radius. Find the ratio of the length of the larger circle's radius to the length of the smaller circle's radius.

A) 2:1 B) 4:1 C) 6:1 D) 9:1 E) NOTA

28. Find the arithmetic mean of the 2013 smallest non-negative integers.

A) 1006.5 B) 1005.5 C) 1007 D) 1006 E) NOTA

29. If $f\left(\sqrt{t+2}\right)=\frac{t+3}{t-1}$, what value is fixed by f? A number a is fixed by function f if $f(a)=a$.

A) -1 B) $1+2\sqrt{2}$ C) $1-2\sqrt{2}$ D) $1+\sqrt{2}$ E) NOTA

30. In ΔABC, point D is on $\overline{BC}$ such that $\overline{AD}$ is an angle bisector of $\angle A$. Point E is on $\overline{AD}$, and point F is on the extension of $\overline{BC}$ such that $\overline{EF}$ is the perpendicular bisector of $\overline{AD}$. Given that $\left|\overline{FC}\right|=4$ and $\left|\overline{FB}\right|=9$, find the value of $\left|\overline{FD}\right|$.

A) 6 B) 8 C) 5 D) 7 E) NOTA