Theta Logarithms & Exponents 2013 MAΘ National Convention

Note: For all questions, answer "(E) NOTA" means none of the above answers is correct.

1. The element C^{14} has a half-life of 5730 years. There is 100 grams of C^{14} in a particular sample now. Which of the following gives the correct amount of C^{14} 11460 years from now?

   (A) 400 grams (B) 50 grams (C) 25 grams (D) 0 grams (E) NOTA

2. A scientist adds a small amount of bacteria to a growth medium on Day 0. On Day 4, the scientist determines that the population of bacteria has quadrupled from its original value. If the population grows exponentially in time, which of the following is closest to the day the scientist can expect the population to be one million times its initial value?

   (A) Day 20 (B) Day 40 (C) Day 1000 (D) Day 10 (E) NOTA

3. What is the units digit of 3^{2013}?

   (A) 1 (B) 3 (C) 7 (D) 9 (E) NOTA

4. If $f(x) = a\left(2^{bx}\right)$, find the value of *b* if $f(0) = 5$ and $f(3) = 20\sqrt{2}$.

   (A) $\frac{7}{3}$ (B) $\frac{5}{3}$ (C) $\frac{\log 20\sqrt{2}}{3}$ (D) $\frac{5}{6}$ (E) NOTA

5. Write an equation in terms of *x* for the area of a rectangle bounded by the graph of $y = e^{-x^2}$ and the *x*-axis. The rectangle is situated so that one of the sides lies along the *x*-axis, the side adjacent to this side is parallel to the *y*-axis, and two adjacent vertices of the rectangle lies on the graph of $y = e^{-x^2}$.

   (A) $A = x \cdot e^{-x^2}$ (B) $A = 2x \cdot 2e^{-x^2}$ (C) $A = 2x \cdot e^{-x^2}$ (D) $A = \left(e^{-x^2}\right)^2$ (E) NOTA

6. Given that $x > 0$, expand completely: $\ln \frac{\left(x^2+3\right)^2}{x \cdot \sqrt[3]{x^2+1}}$

   (A) $2\ln\left(x^2+3\right) - \ln x - \frac{1}{3}\ln\left(x^2+1\right)$ (B) $2\ln\left(x^2+3\right) - \ln x + \frac{1}{3}\ln\left(x^2+1\right)$

   (C) $4\ln x + 2\ln 3 - \ln x - \frac{2}{3}\ln x + \frac{1}{3}\ln 1$ (D) $4\ln x + 2\ln 3 - \ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln 1$ (E) NOTA

7. What is the value of r^3 if $\frac{1}{r} = \sqrt{.01}$?

(A) $\frac{r}{1}$ (B) 10 (C) 100 (D) 1000 (E) NOTA

8. Consider the exponential function *C* below (where $t = 0$ represents the initial time) and choose the correct statement.
$C(t) = 300(.56)^t$

(A) *C* describes exponential growth and the initial amount is 300.
(B) *C* describes exponential decay and the initial amount is 300.
(C) *C* describes exponential growth and the initial amount is .56
(D) *C* describes exponential decay and the initial amount is .56
(E) NOTA

9. Your math teacher has asked you to solve $\log_2 x + \log_2(x+6) = 4$ for *x*. Which of the following equations has the same positive solution(s) as the logarithmic equation?

(A) $x^2 + 6x = 16$ (B) $x^2 + 6x = 4$ (C) $x^2 + x + 6 = 4$ (D) $x^2 + 6x = 8$ (E) NOTA

10. Consider the quadratic function $y = f(x)$ that passes through $(1, \log_2 1), (2, \log_2 2), (4, \log_2 4)$ to be an approximation of $F(x) = \log_2 x$. Estimate the value of $\log_2 3$ by evaluating $f(3)$ to four decimal places.

(A) 1.5850 (B) 1.6667 (C) 1.7321 (D) 3.000 (E) NOTA

11. A few weeks into the SARS (Severe Acute Respiratory Syndrome) epidemic of 2003, the World Health Organization (WHO) determined that the rate of infection was increasing at a rate of 4% per day. Approximately 1800 people had been infected by April 1, 2003. Assuming an exponential model, determine when the WHO could expect the number of infected persons to reach 7200 people.

(A) April 4, 2003 (B)April 18, 2003 (C) May 5, 2003 (D) June 16, 2003 (E) NOTA

12. Evaluate: $\log_{1024}\left(\prod_{j=1}^{100} 4^j\right)$

(A) 0 (B) 10 (C) 101 (D) 1010 (E) NOTA

13. Which of the following exponential equations models your salary *s* (in dollars per hour) over time *t* (in years) given the following condition: You get a job paying $10 per hour. Each year your salary goes up by 5% to keep up with the cost of living.
(A) $s(t) = 10(.05)^t$ (B) $s(t) = 10(1.50)^t$ (C) $s(t) = 10(.105)^t$ (D) $s(t) = 5(1.05)^t$ (E) NOTA

14. Which of the following statement(s) is/are true?

I. $\sum_{n=0}^{100} n^4 = \sum_{n=1}^{100} n^4$ II. $\sum_{j=0}^{100} 2 = 200$ III. $\sum_{k=0}^{100} (2+k) = 2 + \sum_{k=0}^{100} k$

IV. $\sum_{i=1}^{100} (i+1)^2 = \sum_{i=0}^{99} i^2$ V. $\sum_{k=0}^{100} k^3 = \left(\sum_{k=0}^{100} k\right)^3$

(A)I , III, V (B) I (C) I, II, III, IV, V (D) I, V (E) NOTA

15. Calculate the ratio $\frac{x}{y}$ if $2\log_3(x-2y) = \log_3 x + \log_3 y$, where $x > 2y > 0$.

(A) 1 (B) 2 (C) 1 or 4 (D) 4 (E) NOTA

16. Consider $f(x) = \log_b x$ and $g(x) = \log_{1/b} x$ where $0 < b < 1$. Which of the following must be true?

I. The graphs of *y=f(x)* and *y=g(x)* are mirror images of each other with respect to the *x*-axis.

II. The point (1,0) is on the graph of *y=f(x)* and *y=g(x)*.

III. The point (*b*,1) is on the graph of *y=f(x)* and *y=g(x)*.

IV. Given that *x* is a positive real number, the range of *f* and *g* is the set of real numbers.

V. Both functions *f* and *g* are one-to-one.

(A) I, II, IV, V (B) I, II, III, IV (C) II, IV, V (D) I, II, III, IV, V (E) NOTA

17. There exist positive integers *M*, *A*, and *T* with no two of which having common factors greater than 1 such that $M \log_{1568} 7 + A \log_{1568} 2 = T$. What is the value of *M+A+T*?

(A) 9 (B) 2013 (C) 8 (D) 10 (E) NOTA

18. Solve for x: $\sum_{n=1}^{\infty} \log_{2^{(2n)}} x = 8$

(A) 8 (B) $\frac{1}{4}$ (C) 4 (D) 256 (E) NOTA

19. Evaluate: $\sum_{n=1}^{4095} \log_4\left(\frac{n+1}{n}\right)$

(A) 1 (B) 5 (C) 6 (D) 12 (E) NOTA

20. How many real solutions x does $\log_3\left(10-3^x\right)=2-x$ have?

(A) 2 (B) 1 (C) 0 (D) Infinitely many. (E) NOTA

21. How does the graph of $y=\ln(x+2)+3$ compare with the graph of the parent function $y=\ln x$?

(A) Translated up 3 units and right 2 units. (B) Translated up 3 units and left 2 units.
(C) Translated up 2 units and right 3 units. (D) Translated up 2 units and left 3 units.
(E) NOTA

22. In an old television commercial, a woman who liked her shampoo so much that she told two people, and each of them told two people, and each of them told two people, etc. Suppose there are 10 such levels of "telling" and that everyone who hears of the shampoo does use it and does tell two friends. How many shampoo users (including the original woman) would be in this group?

(A) 20 (B) 2047 (C) 2046 (D) 1024 (E) NOTA

23. If $25^x + 25^{-x} = 47$, find the value of $5^x + 5^{-x}$.

(A) 36 (B) 6 (C) $\sqrt{47}$ (D) 47^2 (E) NOTA

24. If $\log_8 3 = k$, express $\log_2 18$ in terms of k.

(A) $6k+3$ (B) $6k+1$ (C) $\frac{k+3}{9}$ (D) $8k-3$ (E) NOTA

25. If $\log 2 \approx .3010$ and $\log 3 \approx .4771$ then which of the following is closest to the value of *x*, given that $2^{x-2} = 225$?

(A) 9.8 (B) 9.0 (C) 15.7 (D) 1.5 (E) NOTA

26. If $2^{x+7} - 3\left(2^x\right) = 5^{y+2} - 9\left(5^y\right)$ and *x* and *y* are integers, determine the value of *xy*.

(A) 36 (B) 16 (C) 9 (D) 12 (E) NOTA

27. How many real solutions *x* does $3\left(4x^2 - 9\right) = \left(4x^2 - 9\right)e^{9-x}$ have?

(A) 0 (B) 1 (C) 3 (D) 5 (E) NOTA

28. If $f(x) = 4^{\log_2 x}$ and $g(x) = 2^{\log_2 5}$ evaluate $f(g(x))$.

(A) $(\log_2 25)^2$ (B) $2\log_2 5$ (C) 25 (D) 1024 (E) NOTA

29. Let $f(x) = x\sqrt{x\sqrt{x\sqrt{\cdots}}}$. Evaluate $\dfrac{\log f(x)}{\log x}$ for *x*-values which $f(x)$ converges.

(A) $\dfrac{3}{2}$ (B) 2 (C) $\dfrac{1}{2}$ (D) 10 (E) NOTA

30. The sum of the first two terms of a seven-term geometric sequence is 20, and the sum of the sum of the last two terms is $-\dfrac{5}{8}$. Find the sum of all seven terms of the sequence.

(A) $\dfrac{215}{8}$ (B) $\dfrac{55}{2}$ (C) $\dfrac{155}{3}$ (D) $\dfrac{165}{8}$ (E) NOTA