1. $\begin{bmatrix} 1+3 & -2+6 \\ 0+5 & 4+-2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 5 & 2 \end{bmatrix}$ A

2. $\begin{bmatrix} 1-2 & -2--2 & 3--1 \\ -4--1 & 5--3 & -6-2 \\ 7-1 & -8--1 & 9-2 \end{bmatrix}^{tr}$
$= \begin{bmatrix} -1 & 0 & 4 \\ -3 & 8 & -8 \\ 6 & -7 & 7 \end{bmatrix}$ C

3. $\begin{bmatrix} 2\cdot1-3\cdot5 & 2\cdot3-3\cdot-2 \\ 2\cdot4-3\cdot-1 & 2\cdot2-3\cdot3 \end{bmatrix}$
$= \begin{bmatrix} -13 & 0 \\ 11 & -5 \end{bmatrix} \Rightarrow$ B
easier way: $2\cdot4-3\cdot5=-7$

4. $4x-3=4 \Rightarrow x=\frac{7}{4}$
$4+3y=7 \Rightarrow y=1$ $\Rightarrow \frac{11}{4}$ D

5. $r-3s=5$ ①
$4r+2s=-1$ ②
4①−②$\Rightarrow -14s=21$
$s=-\frac{3}{2} \Rightarrow r=\frac{1}{2} \Rightarrow$ C

6. $\begin{bmatrix} -1\cdot5+2\cdot1 & -1\cdot3+2\cdot-2 \\ 4\cdot5+-2\cdot1 & 4\cdot3+-2\cdot-2 \end{bmatrix} = \begin{bmatrix} -3 & -7 \\ 18 & 16 \end{bmatrix}$ A

7. Dimensions do not allow multiplication. E

8. $\begin{bmatrix} 2\cdot-3+-2\cdot7+3\cdot1 & 2\cdot0+-2\cdot2+3\cdot-1 \\ 0\cdot-3+4\cdot7+-1\cdot1 & 0\cdot0+4\cdot2+-1\cdot-1 \end{bmatrix}$
$= \begin{bmatrix} -17 & -7 \\ 27 & 9 \end{bmatrix}$ D

9. $\begin{bmatrix} -6+0+-4 & 9+-4+4 & 3+8+4a \\ -2+0+3 & 3+2b-3 & 1-4b-3a \\ 2+0+0 & -3+6+0 & -1-12+0 \end{bmatrix}$
$= \begin{bmatrix} -10 & 9 & 4a+11 \\ 1 & 2b & -3a-4b+1 \\ 2 & 3 & -13 \end{bmatrix}$ B

10. $C=B^{-1}A$
$B^{-1}=\frac{1}{6}\begin{bmatrix} 0 & -3 \\ 2 & 1 \end{bmatrix}$
$C=\frac{1}{6}\begin{bmatrix} 0 & -3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -13 & -3 \\ -10 & -6 \end{bmatrix}$
$=\frac{1}{6}\begin{bmatrix} 30 & 18 \\ -36 & -12 \end{bmatrix}=\begin{bmatrix} 5 & 3 \\ -6 & -2 \end{bmatrix}$
product = 180 D

11. $2\cdot5-3\cdot-2=16$ C

12. $1\cdot\begin{vmatrix} 7 & 4 \\ -3 & 2 \end{vmatrix}+3\cdot\begin{vmatrix} -2 & -3 \\ 7 & 4 \end{vmatrix}$
$26+3\cdot13=65$ C

13. $-1\cdot\begin{vmatrix}4 & 1\\ -2 & a\end{vmatrix}+-2\cdot\begin{vmatrix}b & 1\\ c & a\end{vmatrix}$

$-4a-2-2ba+2c$ C

14. count down, then over B

15. $2x-y=5$
$3x\quad=-2$
$-x-y=7 \Rightarrow -7$ A

16. $-12-3a=-5-14$
$-3a=-7$
$a=\frac{7}{3}$ D

17. $A^{-1}=\frac{1}{|A|}A^{adj}=\frac{1}{2}\begin{bmatrix}4 & 2\\ 1 & 1\end{bmatrix}$

$\text{Sum}=\frac{4+2+1+1}{2}=4$ D

18. non-square matrices aren't invertable E

19. singular means $|A|=0 \Rightarrow$ D

20. ↗ $\Rightarrow\begin{bmatrix}-1 & 3\\ 0 & 2\end{bmatrix}$ C

21. cofactor is the signed determinant of what's left when you get rid of an element's row & column $=+0$ B

22. $\begin{vmatrix}-1 & 4 & 2\\ 3 & 0 & -1\\ 2 & -A & 6\end{vmatrix}=0$

$-4\cdot\begin{vmatrix}3 & -1\\ 2 & 6\end{vmatrix}+A\cdot\begin{vmatrix}-1 & 2\\ 3 & -1\end{vmatrix}=0$

$-80-5A=0 \Rightarrow A=-16$ B

23. $\begin{vmatrix}2A & 4\\ 2 & A\end{vmatrix}=0$

$2A^2-8=0$
$A^2=4 \Rightarrow A=\pm 2$ A

24. $R3\to\begin{vmatrix}0 & 0 & 4 & 2\\ 2R4\to -2 & 0 & 0 & 8\\ 3\cdot 4\to -3 & -1 & 5 & -3\\ 1\cdot 4\to 3 & 1 & -1 & 5\end{vmatrix}$

oops... you can perform manipulations like this, but I made these 3 dependent...

$=4\begin{vmatrix}-2 & 0 & 8\\ -3 & -1 & -3\\ 3 & 1 & 5\end{vmatrix}-2\begin{vmatrix}-2 & 0 & 0\\ -3 & -1 & 5\\ 3 & 1 & -1\end{vmatrix}$

$=4\left(-2\begin{vmatrix}-1 & -3\\ 1 & 5\end{vmatrix}+8\begin{vmatrix}-3 & -1\\ 3 & 1\end{vmatrix}\right)-2\left(-2\begin{vmatrix}-1 & 5\\ 1 & -1\end{vmatrix}\right)$

$=4(4+0)-2(8)=0$

Much faster by calculator, & fewer dumb errors.

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25. swapping rows or columns changes the sign $\Rightarrow$ D

26. $\begin{bmatrix}3&4\\5&6\end{bmatrix}\begin{bmatrix}0&-1\\1&0\end{bmatrix}=\begin{bmatrix}4&-3\\6&-5\end{bmatrix}$

$6+-5=1$ B

27. $\frac{1}{6+a}\begin{bmatrix}2&-a\\m&3\end{bmatrix}\Rightarrow$ B

28. $-4-x(3x-1)>-8$

$0>3x^2-x-4$

$0>(3x-4)(x+1)$

$x=\frac{4}{3}$ $x=-1$, 0 works

$-1<x<\frac{4}{3}$ A

29. I✓ II✓ III✓ IV✓ D

└closure means if you add two matrices, the result is another matrix.

30. I✗ II✓ III✗ IV✗ B

31. $2n+5d=15$

$6n+Jd=120$

$\begin{vmatrix}2&5\\6&J\end{vmatrix}=0=2J-30\Rightarrow$ B

32. $z=\dfrac{\begin{vmatrix}1&3&1\\0&2&-1\\-1&1&1\end{vmatrix}}{\begin{vmatrix}1&3&0\\0&2&1\\-1&1&1\end{vmatrix}}=\dfrac{\begin{vmatrix}2&-1\\1&1\end{vmatrix}-\begin{vmatrix}3&1\\2&-1\end{vmatrix}}{\begin{vmatrix}2&1\\1&1\end{vmatrix}-\begin{vmatrix}3&0\\2&1\end{vmatrix}}$

$=\frac{3--5}{1-3}=\frac{8}{-2}=-4$ D

33. The series which turns M into I will turn I into $M^{-1}\Rightarrow$ C

34. $4\cdot3^3=108$ B

35. Adjoint is transposed matrix of cofactors.

$(A^{adj})_{3,2}=\text{cofactor}(A_{2,3})$

$=-\begin{vmatrix}3&0\\-1&1\end{vmatrix}=-3$ C

36. An eigenvector of a transformation is a vector which points the same direction (or opposite) before & after.

$\begin{bmatrix}1\\1\end{bmatrix}\to\begin{bmatrix}1\\1\end{bmatrix}$✓ $\begin{bmatrix}2\\-3\end{bmatrix}\to\begin{bmatrix}-8\\12\end{bmatrix}$✓

$\begin{bmatrix}-2\\2\end{bmatrix}\to\begin{bmatrix}6\\-10\end{bmatrix}$✗ $\Rightarrow$ ~~XX~~ A

37. An eigenvalue is the ratio by which an eigenvector is "stretched" during transformation. If $\underline{x}$ is an eigenvector of $\bar{A}$ with eigenvalue λ,

$\bar{A}\underline{x} = \lambda\bar{I}\underline{x}$ where $\bar{I}$ is identity

$$|\bar{A} - \lambda\bar{I}| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 2 \\ 1 & -\lambda \end{vmatrix} = 0$$

$\lambda^2 - \lambda - 2 = 0$

$(\lambda - 2)(\lambda + 1) = 0 \Rightarrow \lambda = 2, -1$ D

38. $|A^{-1}| = \frac{1}{|A|}$ guess A immediately

$$|A| = 1\cdot\begin{vmatrix} -3 & 0 & 5 \\ 2 & 1 & 4 \\ -2 & 0 & 2 \end{vmatrix} + 2\begin{vmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ -3 & -2 & 2 \end{vmatrix} + x\begin{vmatrix} 1 & -3 & 0 \\ 0 & 2 & 1 \\ -3 & -2 & 0 \end{vmatrix}$$

~~$= 1\begin{vmatrix} 0 & 5 \\ 1 & 4 \end{vmatrix} + 2\begin{vmatrix} -3 & 0 \\ 2 & 1 \end{vmatrix}$~~

use calc: $|A| = 4 + 2\cdot 78 + x\cdot 11$

$= 11x + 160 \Rightarrow$ A

39.
$$M = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$ A

40. $\begin{cases} d + w + m = 1 \\ w = 3d - 3m \\ d = w - m \end{cases}$

$2w = 1 \Rightarrow w = \frac{1}{2}$ C