**Practice Round Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

P1. Evaluate: $12^2 + 36^2$

P2. Find the area, in square meters, of a triangle with base 6 meters and height 4 meters.

P3. Solve for $x$: $x + 4 = 102$

P4. What is the area of a regular hexagon with side length $2(3)^{1/4}$?

P5. Let $A$, $B$, $C$, and $D$ be the answers to problems P1, P2, P3, and P4, respectively.
Evaluate: $\frac{A}{B} + C - D$

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**Round #1 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

1. Find the largest integer $x$ such that $|x-2| < |x-6|$.

2. The sum of the digits of a four-digit positive integer is 23. The hundreds digit is 3 more than the thousands digit. The tens digit is 5 less than twice the hundreds digit. The units digit is 1 more than twice the thousands digit. Find the largest positive prime factor of this number.

3. The perimeter of a right triangle is $12 + 8\sqrt{3}$. The sum of the squares of the three side lengths is 294. Find the area of the triangle in simplest radical form.

4. If $4^x - 4^{x-1} = 24$, find the value of $(2x)^{2x}$.

5. Let $A$, $B$, $C$, and $D$ be the answers to problems 1, 2, 3, and 4, respectively.
Evaluate: $\frac{B-D}{A} + C^2$

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**Round #2 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

6. How many digits are there in the binary representation of 5566?

7. Find the number of terms in the following arithmetic sequence: $4, 1, -2, \ldots, -32$

8. A *diagonal* of a convex polygon is a line segment whose endpoints are two nonadjacent vertices. How many diagonals can be drawn in a convex polygon with sixteen sides?

9. If $f$ varies jointly as $g^2$ and $h$, and $f = 128$ when $g = 4$ and $h = 2$, find $f$ when $g = 3$ and $h = 6$.

10. Let $A$, $B$, $C$, and $D$ be the answers to problems 6, 7, 8, and 9, respectively.
Evaluate: $\log_{10}(D - C - B + 1)^A$

**Round #2 Theta School Bowl**
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**Round #3 Theta School Bowl**
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11. Let $L(x)$ be a linear function of positive slope and $I(x)$ be the inverse of $L(x)$. Given that $L(x) = 4I(x) + 3$ for all real $x$, find the value of $L(10)$.

12. For integers $x$ and $y$, let $S$ be the set of all points $(x, y)$ in the plane such that $0 \leq x \leq 4$ and $0 \leq y \leq 4$. Find the number of squares all of whose vertices are in $S$.

13. What is the remainder when $4 \cdot 3^{2013}$ is divided by 10?

14. If $f(x) = 3x - 4$ and $g(x) = \sqrt{x+2}$, evaluate: $f(g^{-1}(5))$

15. Let $A$, $B$, $C$, and $D$ be the answers to problems 11, 12, 13, and 14, respectively. Evaluate: $BD^C + A$

**Round #3 Theta School Bowl**
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**Round #4 Theta School Bowl**
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16. Solve for $x$: $\frac{x+3}{x+5} = \frac{x+1}{x+2}$

17. Find the value of $3n$, where $\binom{n}{1}^3 + 3\binom{n}{3} + 6\binom{n}{0}\binom{n}{1}\binom{n}{2} = 84$.

18. The determinant of a $2 \times 2$ matrix $A$ is 2, and the determinant of a $3 \times 3$ matrix $B$ is $-3$. Evaluate: $\det(2A)\det(3B)$.

19. What is the product of the positive solutions to $x^3 - 7x + 6 = 0$?

20. Let $A$, $B$, $C$, and $D$ be the answers to problems 16, 17, 18, and 19, respectively. Evaluate: $B^{-1}C(2A - D)$

**Round #4 Theta School Bowl**
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**Round #5 Theta School Bowl**
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21. Find the sum of the third hexagonal number and the fourth octagonal number.

22. The base-10 number 360 is converted to an integer in base $b$, where $b$ is an integer from 2 to 20, inclusive. Afterwards, the product of the digits of the resulting number is calculated and expressed in base 10. Let $f(b)$ equal the result of this procedure. For example, if $b = 13$, $360 = 219_{13}$, so that $f(13) = (2)(1)(9) = 18$. Find the maximum possible value of $f$.

23. Find the smallest possible value for $y$ if $y = x^2 + 4x + 9$ and $x$ is a real number.

24. Let $k$ be the smallest number greater than 1 such that $\log_k 3 \leq 4$. Find the value of $k^8$.

25. Let $A$, $B$, $C$, and $D$ be the answers to problems 21, 22, 23, and 24, respectively.
Evaluate: $A/C + B/D^2$

**Round #5 Theta School Bowl**
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Evaluate: $A/C + B/D^2$

**Round #6 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

26. Let vectors $\mathbf{a} = [-1, 0, 1]$, $\mathbf{b} = [3, 4, 3]$, and $\mathbf{c} = [2, -3, 2]$. If $[-6, -17, 6] = c_1\mathbf{a} + c_2\mathbf{b} + c_3\mathbf{c}$, find the value of $c_1c_2c_3$.

27. Find the *negative* integer $x$ closest to 0 such that $x \equiv 1 \pmod 3$, $x \equiv 3 \pmod 5$, and $x \equiv 5 \pmod 7$.

28. A collection of positive integers, not necessarily distinct, add up to 31. Let $P$ be the product of all the elements in the collection. Let $Q$ be the largest possible value of $P$ among all such collections. Find the sum of the digits of $Q$.

29. Find the area of the region in the plane bounded by the graphs of $(x-3)^2 + (y-12)^2 = 144$ and $(x+4)^2 + (y+12)^2 = 169$.

30. Let $A$, $B$, $C$, and $D$ be the answers to problems 26, 27, 28, and 29, respectively.
Evaluate: $|A| + |B| + |C| + |D|$

**Round #6 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

26. Let vectors $\mathbf{a} = [-1, 0, 1]$, $\mathbf{b} = [3, 4, 3]$, and $\mathbf{c} = [2, -3, 2]$. If $[-6, -17, 6] = c_1\mathbf{a} + c_2\mathbf{b} + c_3\mathbf{c}$, find the value of $c_1c_2c_3$.

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Evaluate: $|A| + |B| + |C| + |D|$

**Round #7 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

31. In the Cartesian plane, let $O$ be the origin, $Q = (5, 0)$ and $P$ a point on the circle with equation $x^2 + y^2 = 36$. As $P$ goes along the entire circumference of the circle, the centroid of triangle $OPQ$ traces out a curve $C$. Find the area enclosed by $C$.

32. A jar contains an equal number of black marbles and white marbles. Marbles of the same color are indistinguishable from each other. Three marbles are randomly drawn from the jar without replacement. The probability that all the marbles drawn are white is 1/12. How many total marbles are in the jar?

33. If $x + y = 8$ and $x^2 + y^2 = 232$, find $|x| + |y|$.

34. Function $f$ has $f(4096^2) = -30$, $f(8^4) = 100$, $f(81^3) = -51$, and $f(25^6) = 12$. If the domain of $f$ is $D = \{4096^2, 8^4, 81^3, 25^6\}$ and $m$ is the smallest element in $D$, find $f(m)$.

35. Let $A$, $B$, $C$, and $D$ be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $A^{-1}C\pi \log_B D$

**Round #7 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

31. In the Cartesian plane, let $O$ be the origin, $Q = (5, 0)$ and $P$ a point on the circle with equation $x^2 + y^2 = 36$. As $P$ goes along the entire circumference of the circle, the centroid of triangle $OPQ$ traces out a curve $C$. Find the area enclosed by $C$.

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35. Let $A$, $B$, $C$, and $D$ be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $A^{-1}C\pi \log_B D$

**Round #8 Theta School Bowl**
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36. Given that $M = \begin{bmatrix} -29 & -20 \\ 42 & 29 \end{bmatrix} = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$, find the sum of the entries of $M^{10}$.

37. What is 68 more than one-fourth of 6868?

38. What is the least common multiple of 135 and 84?

39. Find the sum of the digits of $(10-1)(10^2-1)(10^4-1)(10^8-1)$ when written in base 10.

40. Let $A$, $B$, $C$, and $D$ be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $(BCD)^{A-2}$

**Round #8 Theta School Bowl**
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Evaluate: $(BCD)^{A-2}$

**Round #9 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

41. In convex quadrilateral $ABCD$, $|AB| = \sqrt{6}$, $|BC| = 5 - \sqrt{3}$, $|CD| = 6$, $m\angle ABC = 135°$, and $m\angle BCD = 120°$. Find the length of $AD$ in simplest radical form.

42. Find the sum of the cubes of the roots of $f(x) = x^2 + 8x - 1$.

43. If $5x - 4 = 16$, find $10x - 14$.

44. The determinant of $M = \begin{bmatrix} -1 & -2 & -5 \\ 0 & -1 & 4 \\ 4 & x & 5 \end{bmatrix}$ is $4x - 47$. If the entry in the second row, second column of $M^{-1}$ is equal to 15, find the value of $x$.

45. Let $A$, $B$, $C$, and $D$ be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $\sqrt{|10(A^2 - C) + B|} + D$

**Round #9 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

41. In convex quadrilateral $ABCD$, $|AB| = \sqrt{6}$, $|BC| = 5 - \sqrt{3}$, $|CD| = 6$, $m\angle ABC = 135°$, and $m\angle BCD = 120°$. Find the length of $AD$ in simplest radical form.

42. Find the sum of the cubes of the roots of $f(x) = x^2 + 8x - 1$.

43. If $5x - 4 = 16$, find $10x - 14$.

44. The determinant of $M = \begin{bmatrix} -1 & -2 & -5 \\ 0 & -1 & 4 \\ 4 & x & 5 \end{bmatrix}$ is $4x - 47$. If the entry in the second row, second column of $M^{-1}$ is equal to 15, find the value of $x$.

45. Let $A$, $B$, $C$, and $D$ be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $\sqrt{|10(A^2 - C) + B|}$+D

**Round #10 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

46. If $f(x) = \log_2\left(1 + \sqrt{8x+1}\right) - 2$, evaluate: $f(6) + f(28) + f(496) + f(8128) + f(33550336)$

47. The first term of an arithmetic sequence is $-40$ and the third term is $-24$. What is the smallest positive term of this sequence?

48. Find the sum of the coefficients when $(6x - 2y + z)^5$ is fully expanded and like-terms combined.

49. How many 4-digit positive integers in base 10 have 5 as a digit immediately followed by a 3 (as the next digit)?

50. Let $A$, $B$, $C$, and $D$ be the answers to problems 46, 47, 48, and 49, respectively.
Evaluate: $D \log_B\left(\frac{A+C}{25} + 2\right)$

**Round #10 Theta School Bowl**
**Mu Alpha Theta National Convention 2013**

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48. Find the sum of the coefficients when $(6x - 2y + z)^5$ is fully expanded and like-terms combined.

49. How many 4-digit positive integers in base 10 have 5 as a digit immediately followed by a 3 (as the next digit)?

50. Let $A$, $B$, $C$, and $D$ be the answers to problems 46, 47, 48, and 49, respectively.
Evaluate: $D \log_B\left(\frac{A+C}{25} + 2\right)$