Practice Round Theta State Bowl
Mu Alpha Theta National Convention 2013

P1. Determine the slope of the line with equation $15x + 3y = 100$.

P2. Find the smallest positive integer n that makes $\left(\sqrt{2}\right)^n$ a rational number.

P3. The acute angle supplementary to 134° is x degrees. Find x.

P4. Evaluate: $2 \ln e^{2013}$

P5. Let A, B, C, and D be the answers to problems P1, P2, P3, and P4, respectively.
Evaluate: $D(A + 5) + BC$

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Round #1 Theta State Bowl
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1. Find x as a common fraction: $4 + \sqrt{10 - x} = 6 + \sqrt{4 - x}$

2. Regular octagon $ABCDEFGH$ has $A = (0, 0)$ and $D = (3, 4)$. Find the perimeter of the octagon. Express your answer in the form $x\sqrt{y} + z$, where x, y, and z are integers and y is a positive prime number.

3. What is the coefficient of the x^9-term when $(-4 - x)^{11}$ is fully expanded and like-terms combined?

4. Solve for x: $\log_2(2x) + \log_4 x + \log_8 x = 12$

5. Let A, B, C, and D be the answers to problems 1, 2, 3, and 4, respectively.
   Evaluate: $\frac{3(B^2+80B)C}{80AD}$

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   Evaluate: $\frac{3(B^2+80B)C}{80AD}$

Round #2 Theta State Bowl
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6. For integer n, let $\tau(n)$ equal the number of positive divisors of n. How many positive integers N less than 200 satisfy the congruence $\tau(N) \equiv 1 \pmod 2$?

7. Find the area of an ellipse with equation $25(x+20)^2 + 9(y-13)^2 = 450$.

8. A sequence is recursively defined by $a_5 = -35$ and for integers $n > 0$, $a_{n+1} = 2a_n + 3$. Find the value of a_1.

9. What is the remainder when $2x^{603} - 3x^{250} + 10 - 6x^{25}$ is divided by $x + 1$?

10. Let A, B, C, and D be the answers to problems 6, 7, 8, and 9, respectively.
Evaluate: $\frac{(A-C+D)\pi}{B}$

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Round #3 Theta State Bowl
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11. Find, as a common fraction, the sum of all *real numbers* x such that $2x^3 + x^2 - 4 = 8x$.

12. Consider a sequence $a_1, a_2, a_3, \ldots, a_{10}$ of ten consecutive integers. If $a_3 + a_4 = 47$, then find the value of $\sum_{n=1}^{10} a_n$.

13. Find the product of all negative integer values of p such that $P(x) = 4x^2 + 4px + 4 - 3p$ does *not* have two distinct real roots.

14. What is the total surface area of a regular octahedron of volume $4/3$?

15. Let A, B, C, and D be the answers to problems 11, 12, 13, and 14, respectively.
Evaluate: $\left(B - \frac{AD^2}{C}\right)^{\frac{1}{2}}$

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Round #4 Theta State Bowl
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16. How many integers x satisfy $\big||x| - 7\big| \leq 8$?

17. For integers x and y, let S be the set of all points (x, y) in the plane such that $0 \leq x \leq 3$ and $0 \leq y \leq 2$. Let D be the set of all possible distances between two distinct points in S. What fraction of the elements in D are irrational numbers? Express your answer as a common fraction.

18. If a, b, c, and d are nonnegative integers, find the number of solutions to the equation $a + b + c + d = 7$.

19. Find the product of all distinct complex numbers z such that $z^6 = -64$ has a positive real part.

20. Let A, B, C, and D be the answers to problems 16, 17, 18, and 19, respectively.
Evaluate: $A + \frac{C}{BD}$

Round #4 Theta State Bowl
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Round #5 Theta State Bowl
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21. Find the area of a quadrilateral with side lengths of 39, 52, 25, and 60 in that order.

22. Evaluate the following, expressing your answer as a common fraction:
$$-\frac{2}{5}+\frac{5}{25}+\frac{12}{125}+\cdots+\frac{7n-9}{5^n}+\cdots$$

23. What is the sum of the positive divisors of 1352?

24. If M and N are positive perfect cubes less than 1000 such that $M-N=169$, find $M^{\frac{1}{3}}+N^{\frac{1}{3}}$.

25. Let A, B, C, and D be the answers to problems 21, 22, 23, and 24, respectively.
Evaluate: $(A-C)(B^{-1}+D)$

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Evaluate: $(A-C)(B^{-1}+D)$

Round #6 Theta State Bowl
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26. Let M be a 4×4 matrix such that $M \times \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} b \\ c/2 \\ 3d \\ a/4 \end{bmatrix}$ for all real numbers a, b, c, and d. Find the sum of the elements of $3M^{-1}$.

27. If two of the roots of $f(x) = 2x^3 - 3x^2 + ax + b$ are 3 and -2, and a and b are rational, find $|a| + |b|$.

28. Let x and y be positive numbers such that $\log_9 x = \log_{12} y = \log_{16}(x + y)$. If $= y/x$, then find the value of $N^2 - N$.

29. Find the distance from (0,0) to the focus of the parabola with equation $8x + y^2 = 6y - 25$.

30. Let A, B, C, and D be the answers to problems 26, 27, 28, and 29, respectively.
Evaluate: $(B - C)^2 + (A + D)^{\frac{2}{3}}$

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Round #7 Theta State Bowl
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31. Find $P(100)$, where $P(x)$ is a polynomial with real coefficients and $P(x^2) + 2x^2 + 10x = 2xP(x+1) + 3$ for all real x.

32. Solve for x, expressing your answer as a common fraction: $4^{5x-3} = 64^{7x+1}$

33. Let L be a common external tangent of two circles with radii lengths of 5 and 12. The distance between the centers of the two circles is 25. Find the length of the segment in L whose endpoints are the tangency points of the two circles.

34. The sequence $17, 20, 25, 32, \ldots$ has nth term given by $a_n = n^2 + 16$. Find the largest possible value of the greatest common divisor of two consecutive terms of this sequence as n ranges across the positive integers.

35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $A + BC + D$

Round #7 Theta State Bowl
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35. Let A, B, C, and D be the answers to problems 31, 32, 33, and 34, respectively.
Evaluate: $A + BC + D$

Round #8 Theta State Bowl
Mu Alpha Theta National Convention 2013

36. In triangle ABC with centroid P, let D and E be the foot of the medians to sides BC and AC, respectively. If AP is perpendicular to BE, $|AD| = 6$, and $|BE| = 9$, find the area of ABC.

37. If $8100 = 108^a 45^b 50^c$, where a, b, and c are rational numbers, what is the value of b? Express your answer as a common fraction.

38. How many ordered pairs of positive integers (x, y) exist such that the least common multiple of x and y is 100?

39. Let a be a sequence such that $a_1 = 2$ and $a_n(1 - a_{n+1}) = 1$ for $n \geq 1$. Evaluate: $\sum_{n=1}^{833} a_n$

40. Let A, B, C, and D be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $BD + A - C$

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40. Let A, B, C, and D be the answers to problems 36, 37, 38, and 39, respectively.
Evaluate: $BD + A - C$

Round #9 Theta State Bowl
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41. Define $\Pi(S)$ as the product of the elements of a set S. Let $S_1, S_2, S_3, \ldots, S_{15}$ be the nonempty subsets of $S = \{1, 2, 3, 4\}$. Evaluate: $\sum_{n=1}^{15} \left(\Pi(S_n)\right)^{-1}$

42. The set $S = \{x, 18, 4, 13, 6\}$ has an arithmetic mean of 10. Find the median of S.

43. The two primes in-between 50 and 60 are a and b, where $a < b$. Evaluate: $\frac{1}{4}(b^2 - a^2)$

44. Let P be a point inside square $ABCD$ such that $|AP| = 5$, $|BP| = 2\sqrt{2}$, and $|CP| = 3$. Find the area of $ABCD$.

45. Let A, B, C, and D be the answers to problems 41, 42, 43, and 44, respectively.
Evaluate: $\sqrt{A} - \sqrt{B} + \sqrt{C + 1} - \sqrt{D - 4}$

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Evaluate: $\sqrt{A} - \sqrt{B} + \sqrt{C + 1} - \sqrt{D - 4}$

Round #10 Theta State Bowl
Mu Alpha Theta National Convention 2013

46. Find the sum of all positive integers n such that $\frac{2210}{(3n+5)(2n+3)}$ is an integer.

47. Find $x^2 + y^2$ if $x + y = 6$ and $4x - y = 14$.

48. What is the smallest positive value of n to guarantee that a collection of n integers, not necessarily distinct, have two elements with the same units digit?

49. Let $f(x)$ denote the integer closest to $\sqrt{x}$. Evaluate: $\sum_{n=1}^{650} \frac{1}{f(n)}$

50. Let A, B, C, and D be the answers to problems 46, 47, 48, and 49, respectively. Evaluate: $(A + B)^{\frac{1}{3}} + (C + D + 3)^{\frac{1}{2}}$

Round #10 Theta State Bowl
Mu Alpha Theta National Convention 2013

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48. What is the smallest positive value of n to guarantee that a collection of n integers, not necessarily distinct, have two elements with the same units digit?

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