Mu Alpha Theta National Convention: Denver, 2001

State Bowl – Theta Division

1. What is the units digit of $16^{16^{16}}$?

2. What is the sum of the complex roots of the equation $3x^2 - 2x + 1 = 0$?

3. What is the length of the major axis of the ellipse with equation $\frac{(x-2)^2}{49} + \frac{(y+3)^2}{25} = 1$?

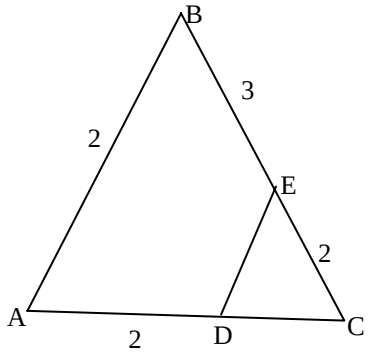
4. What is the magnitude of the vector $-2\vec{i} + 4\vec{j} - 4\vec{k}$?

5. A man has a square plot of ground with area 625 square meters. If the man builds a fence around his property with posts every meter, what is the total number of posts used?

6. Evaluate: $\dfrac{3^4 \times 4^3}{2^5}$

7. Express 114_{10} in base five.

8. What is the length of $\overline{DE}$ in the figure shown if it is parallel to $\overline{AB}$?



9. Simplify: $\left(1-i\sqrt{3}\right)^6$

10. In how many zeros does 148! end when expressed in base 11?

11. What is the smallest natural number with exactly eight positive integral factors?

12. A man purchases a pair of shoes from a merchant for thirty dollars using a fifty-dollar bill. After the man has left with his change, the merchant discovers that the bill is counterfeit. If the shoes cost the merchant twelve dollars, what is the total loss to the merchant, in dollars?

13. What is the smallest natural number greater than one that is a perfect square and a perfect fifth power?

14. Determine the value of x in the following set of equations.

$$x + y = 7$$
$$y + z = 3$$
$$x + z = 5$$

15. What is the smallest natural number that cannot be the number of intersections between a parabola and an ellipse?

16. The fourth and thirteenth terms of an arithmetic sequence are 37 and 100, respectively. What is the twentieth term of this sequence?

17. If $3x + y = 7$ and $x - 2y = 18$, what is the value of y?

18. If $n(q) = \sqrt{2q+3}$ and $p(n(q)) = 4q^2 + 12q + 86$, what is the value of $p(2)$?

19. What is the cosine of the smaller angle between the vectors $3\vec{i} - 2\vec{j}$ and $-2\vec{i} + \vec{j}$?

20. Simplify: $\dfrac{1+3i}{2+4i}$

21. Given that $k(m) = 12m - 13$, what is $k^{-1}(-22)$?

22. Find the sum of all positive integral multiples of three less than one hundred.

23. Determine the radius of the circle with equation $x^2 + 4x + y^2 - 8y = 61$.

24. How many distinct arrangements are there of the letters in the word “illicit”?

25. Simplify: $\log_b a^3 \log_{a^2} b$

26. A grocery store has 40 crates of apples. If each crate contains at least 30 apples and no more than 42 apples, what is the greatest integer n such that there must be at least n crates with the same number of apples as one another?

27. Determine the sum of the twenty smallest natural numbers with an even number of positive integral factors.

28. In a regular n-gon, 252 diagonals can be drawn. What is the value of n?

29. What is the sum of all the positive integral multiples of four between 20 and 100, inclusive?

30. An infinite geometric series has a common ratio of one-third, and its third term is 18. What is the sum of all the terms in the series?

31. If it is windy, there is a 25% chance it will rain. If it is rainy, there is a 20% chance it is also windy. The probability that it is neither windy nor rainy is $\frac{1}{2}$. What is the probability that it is both windy and rainy?

32. Evaluate: $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

33. How many pure imaginary roots does the equation $x^3 + 2x^2 + 3x + 6 = 0$ have?

34. Given that $\log 5 = a$, express $\log 2$ in terms of *a*.

35. What is the probability that when a rook and king of opposite colors are randomly placed on a chessboard, the king is in "check"? Note: this will occur if the rook and king are in either the same column or the same row as one another on the 8 by 8 chessboard.

36. A woman is passing a gumball machine with her three sons, the younger two of which want the same color gumball their big brother gets. If there are four colors in the gumball machine, and there are a total of 1500 gumballs, what is the smallest number she must be willing to purchase to be sure of satisfying them?

37. A boy and a girl race 100 meters, and the girl wins when the boy has only traveled 92 meters. To make the next race more even, the girl starts eight meters behind the starting line of the 100-meter stretch. How many meters behind the winner of the second race will the loser be?

38. Solve for f: $8^{f-2}4^{2f+1} = 16^{3-2f}$

39. Evaluate: $\sum_{n=1}^{15}\left(n^2+3\right)$

40. For what real values of k does $2x^2 + kx - 3 = 0$ have two real roots?

41. The sum of the first n natural numbers is 253. Determine n.

42. What is the maximum possible value of the median of seven whole numbers between zero and 100 inclusive which have a mean of 75?

43. Evaluate: $\log_4 27 \times \log_9 32$

44. A spider eats three flies a day. Until he has done so he has a 50% chance of eating any fly that passes by, but after having eaten three flies it will allow any others to pass unmolested. What is the probability that the fourth fly to attempt to pass will be eaten?

45. Shawn fears that he does not have quite enough soda pop for the guests at his big party. He knows that he can take twelve "empty" cans and drain the dregs to produce one full can and eleven truly empty cans that are of no further use to him. So, he sneaks around his party collecting "empties" and producing new full cans. If he initially had 200 full cans, how many full cans (including his initial 200) can he supply to his guests?

46. Two people start walking in the same direction from the same point on a circular track two kilometers in circumference. If their walking speeds are three and five meters per second, how many seconds ($t > 0$) after they start will they be at the same point on the track as one another for the first time?

47. What is the area of an equilateral triangle whose inscribed circle has an area of 25π?

48. Iris and Marissa are competing in Chalk Talk against seven other contestants. If the judges decide that the competition is too close and therefore decide to award the awards randomly, what is the probability that Iris places in the top three, but is beaten by Marissa?

49. A box contains four red cubes and eight blue cubes. What is the probability that when two cubes are drawn, they are the same color?

50. In triangle *ABC*, $\overline{BD}$ is the bisector of angle *B*, and *D* lies on $\overline{AC}$. If $AB = 6$, $BC = 4$, and $AC = 8$, what is the length of $\overline{AD}$?

51. What is the sum of the mode and median of the following data set?

1, 3, 8, 2, 4, 7, 3, 6, 5, 2, 4, 5, 2

52. Chords $\overline{AB}$ and $\overline{CD}$ of circle *O* intersect at *E* such that $AE = 6$, $EB = 2$, and $CE = 4$. Also, $AC = 7$. What is the length of $\overline{BD}$?

53. In my bathtub, the bath and shower pipes are independent of one another. Filling the tub using only the bath spigot takes ten minutes, so I usually turn on the shower, too, decreasing the filling time to six minutes. How many minutes would it take to fill the tub using only the shower? Note: the drain is closed when I fill the bathtub.

54. How many natural numbers are factors of 33,600?

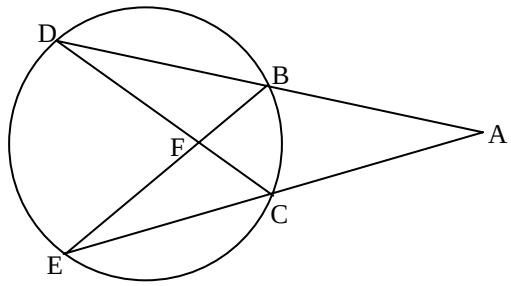
55. Consider the constraints $x - y > 4$ and $y > \frac{3}{5}x + 2$. The points satisfying both of these constraints lie in one or more quadrants. Determine the sum of the numbers of the quadrants in which these points lie.

56. George's coin collection consists entirely of nickels and quarters. The total number of coins in his collection is 32, and the total value of his collection is \$4.60. How many quarters are in his collection?

57. For what value(s) of k will the following system of equations have no solutions?

$$(k+1)x + y = 7$$
$$4x + (k-2)y = 3$$

58. What is the measure of angle *BAC* in the figure if the measure of angle *BFD* is 40° and the measure of arc *DCE* is 170°?



59. You have two bags of marbles. Bag A contains three red and five blue marbles. Bag B contains four red and three green marbles. If a marble is chosen at random from Bag A and placed in Bag B, and a marble is then chosen at random from Bag B, what is the probability that the marble chosen from Bag B is red?

60. What is the sum of the first ten terms of a geometric sequence with first term 144 and second term 72?