For all problems, answer choice E, NOTA, denotes "none of the above." Good luck!

1. What is the sum of all non-real solutions to the equation $x^{2023} = 1$?

   A. -2 B. -1 C. 0 D. 1 E. NOTA

2. Amy is going to her favorite restaurant to get some of her favorite foods. It costs Amy \$9 for her to order 2 grilled cheese and 1 mac and cheese. It costs her \$24.25 to get 5 grilled cheese and 3 mac and cheese. How much would it cost her to get 8 grilled cheese and 5 mac and cheese?

   A. \$38.25 B. \$39.25 C. \$39.50 D. \$41.75 E. NOTA

3. How many integer solutions are there to $x^2 - 8x + 15 \geq 0$

   A. 0 B. 1 C. 2 D. 3 E. NOTA

4. For $0 \leq \theta < 2\pi$, what is the sum of solutions for which $\cos(2\theta) = \sin(\theta)$.

   A. π B. $\frac{3\pi}{2}$ C. $\frac{5\pi}{2}$ D. 3π E. NOTA

5. What is the sum of the coefficients of the expansion of $(2x + y)^5$

   A. 63 B. 81 C. 223 D. 243 E. NOTA

6. In quadrilateral ABCD, $AB = 11$, $BC = 9$, $CD = 23$, $AD = 5$. If the length of diagonal AC is an integer, what is the length of diagonal AC?

   A. 18 B. 19 C. 20 D. 21 E. NOTA

7. Let $\alpha = \frac{5\pi}{12}, \beta = \frac{4\pi}{9}, \gamma = \frac{7\pi}{15}$. Let $f(x) = (\cos(x))^{\csc(x)}$Which of the following inequalities is true?

   A. $f(\alpha) > f(\beta) > f(\gamma)$ B. $f(\gamma) > f(\beta) > f(\alpha)$
   C. $f(\beta) > f(\alpha) > f(\gamma)$ D. $f(\alpha) > f(\gamma) > f(\beta)$ E. NOTA

8. What is the sum of the distinct solutions of $x^5 - 9x^4 + 23x^3 - 11x^2 - 24x + 20 = 0$?

A. 7 B. 8 C. 9 D. 12 E. NOTA

9. Let a and b be 2 distinct real numbers such that $a^2 + 4a + 2 = b^2 + 4b + 2 = 0$. What is $\frac{a}{b} + \frac{b}{a}$?

A. -8 B. -6 C. 6 D. 8 E. NOTA

10. Let x and y be real numbers such that $2 < \frac{x-3y}{x+3y} \leq 13$. Given that $\frac{x}{y}$ is an integer, what is the sum of all distinct values of $\frac{x}{y}$?

A. -39 B. -30 C. -15 D. -9 E. NOTA

11. What is the minimum value of $\frac{4x^2\cos^2(x)+25}{2x\cos(x)}$ for $\frac{3\pi}{2} < x < \frac{5\pi}{2}$?

A. 5 B. $5\sqrt{2}$ C. 10 D. 20 E. NOTA

12. Given the following system of equations

$$4x + 8y + 12z = 1$$
$$3x + 2y + 6z = 0$$
$$5x + 4y - 3z = 8$$

What is the value of x?

A. $-\frac{7}{12}$ B. $\frac{5}{8}$ C. $\frac{3}{4}$ D. No Solution E. NOTA

13. What is the value of $\sqrt{3 + 2\sqrt{3 + 2\sqrt{3+\ldots}}}$?

A. -1 B. 1 C. 2 D. 3 E. NOTA

14. Given a cubic polynomial, $f(x)$ with the following values:

x	1	2	3	4
$f(x)$	-4	-2	4	20

What is $f(5)$?

A. 6 B. 27 C. 46 D. 52 E. NOTA

15. There exists a line $y = ax + b$ that is tangent to the function $y = x^2 - 4x + 5$ at the point $(3,2)$. What is $a + b$?

A. -2 B. 1 C. 2 D. 6 E. NOTA

16. When solving over complex numbers, how many distinct values of y satisfy the following two equations?

$$\frac{y(x+y)}{x^2} = \frac{9}{4}$$
$$x^2y(x+y) = 36$$

A. 0 B. 2 C. 4 D. 8 E. NOTA

17. Two distinct numbers, x, y are chosen randomly from the set $\{3^1, 3^2, 3^3, \ldots, 3^{15}, 3^{16}\}$. What is the probability that $\log_y(x^2)$ is an integer?

A. $\frac{1}{5}$ B. $\frac{7}{30}$ C. $\frac{4}{15}$ D. $\frac{3}{10}$ E. NOTA

18. There are 3 vectors, a, b, c such that $|a| = 2, |b| = 8, |c| = 5$. What is the maximum value of $|3a + 2b + c|$?

A. 15 B. 24 C. 27 D. No maximum E. NOTA

19. What is the remainder of $x^{75} - 25x^{73} + 13$ when divided by $x^2 - 4x - 5$

A. $-4x + 33$ B. $33x - 4$ C. $4x - 33$ D. $4x + 33$ E. NOTA

20. Which of the following vector equations represents the line of intersection between the planes with equations $4x - 2y - 3z = 3$ and $3x + y + 4z = 1$?

A. $\langle 1,2,1\rangle + \langle 1,5,-2\rangle d$ B. $\langle 1,2,1\rangle + \langle -2,5,-1\rangle d$
C. $\langle 1,2,-1\rangle + \langle 1,5,-2\rangle d$ D. $\langle 1,2,-1\rangle + \langle -2,5,1\rangle d$ E. NOTA

21. What is the value of $\cos\left(\frac{2\pi}{9}\right) + \cos\left(\frac{4\pi}{9}\right) + \cos\left(\frac{6\pi}{9}\right) + \cos\left(\frac{8\pi}{9}\right)$

A. $-\frac{1}{2}$ B. 0 C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. NOTA

22. If r_1, r_2, r_3 are roots of the equation

$$x^3 - 3x^2 + 2x - 1 = 0$$

Compute $r_1^3 + r_2^3 + r_3^3$.

A. 11 B. 12 C. 13 D. 14 E. NOTA

23. There are 4 points A, B, C, D such that $AB = 5, BC = 4, CD = 3$. What is the maximum value of BD?

A. 7 B. 8 C. 9 D. 12 E. NOTA

24. How many of the integers in the set {1,2,3,12,72,84,108,144} can be expressed as integer linear combinations of 252 and 612?

A. 2 B. 4 C. 6 D. 8 E. NOTA

25. Compute

$$\sum_{n=6}^{\infty} \frac{1}{n^2 - 8n + 15}$$

A. $\frac{1}{2}$ B. $\frac{3}{4}$ C. 1 D. $\frac{3}{2}$ E. NOTA

26. Given that, $z^2 + \frac{1}{z^2} = 1$, what is the non-negative value of $z^{2023} + \frac{1}{z^{2023}}$?

A. 0 B. 1 C. $\sqrt{2}$ D. $\sqrt{3}$ E. NOTA

27. a_n is an arithmetic sequence and b_n is a geometric sequence. Let $c_n = a_n + b_n$. Given $c_0 = 4, c_1 = 6, c_2 = 9, c_3 = 15$, compute c_4.

A. 30 B. 32 C. 34 D. 36 E. NOTA

28. What is the sum of the prime factors of $9^4 + 4^5$?

A. 54 B. 83 C. 447 D. 1522 E. NOTA

29. Given, $2(a^2 + b^2) = c^2 + d^2$, the maximum value of the expression

$$\frac{30bc + 6bd - 30ad + 6ac}{c^2 + d^2}$$

can be expressed as $\sqrt{n}$ for some integer n. What is n?

A. 234 B. 468 C. 936 D. 1872 E. NOTA

30.

$$a^2 + ab + b^2 = 49$$
$$a^2 + ac + c^2 = 64$$
$$b^2 + bc + c^2 = 9$$

What is the largest value of $a + b + c$?

A. -5 B. 3 C. 5 D. 9 E. NOTA