Circumference/Perimeter/Area/Volume Solutions

Mu Alpha Theta National Convention 2003

1. A Perimeter must be divisible by 3.

2. B altitude to hypotenuse = $\sqrt{6 \bullet 8} = 4\sqrt{3}, area = \frac{1}{2} \bullet 4\sqrt{3} \bullet 4$, $28\sqrt{3}$

3. C Let x = radius of smaller circle. Shorter base = 2x. Radius of larger circle is 8-x. Distance between centers (altitude) is 8. Area = $\frac{1}{2}(2x + 16 - 2x) \bullet 8 = 64$

4. C Triangle is right. smaller leg: radius of circle is $3\sqrt{2}$ so leg is $6\sqrt{2}$,longer leg: radius of circle is $4\sqrt{2}$ so leg is $8\sqrt{2}$. area = $\frac{1}{2} \bullet 6\sqrt{2} \bullet 8\sqrt{2} = 48.$

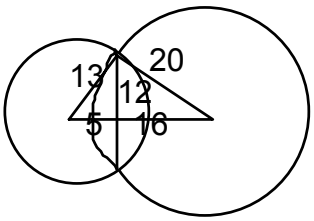
5. D Distance from center is perpendicular bisector of the chord. Forms a right triangle with legs 8 and 15, hypotenuse 17. Circumference = 34π .

6. C Let TW=x and WX=2x and y= TV. $6 = \frac{1}{2}xy, 12 = xy$, area of rectangle is $3x \bullet y = 3 \bullet 12 = 36$

7. D diagonal of one face = $\sqrt{6}$, edge of cube = $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$, V = $(\sqrt{3})^3 = 3\sqrt{3}$

8. A side of square = 6 so area =36, rectangle = $x \bullet 4x = 36,$ making x = 3, w=3, l=12, p=30

9. B Side of square =8, area =64,radius of circle =4, area = 16π , $64 - 16\pi$



10. B
Using Pythagorean theorem, other radius is 20 so C= 40π

11. A Edge of cube = 6, diagonal of cube = $6\sqrt{3}$, radius of sphere is one half diagonal of cube = $3\sqrt{3}$, $\frac{4}{3}\pi(3\sqrt{3})^3 = 108\sqrt{3}$

12. D Drawing diagonals and forming 4 right triangles with legs 4 and $4\sqrt{3}$, hypotenuse which is side of rhombus has length 8. p=32, area = $\frac{1}{2} \bullet 8 \bullet 8\sqrt{3} = 32\sqrt{3}$, product = $1024\sqrt{3}$

13. A To find central angle measure, $20\pi = \frac{x}{360} \bullet 100\pi, x = 72°,$ so sector is $\frac{1}{5}$ of the circle. C= $20\pi, \frac{1}{5} \bullet 20\pi = 4\pi$

14. D diagonal of cube =18, edge of cube = $\frac{18}{\sqrt{3}} = 6\sqrt{3}$, area of a face = 108

15. A Altitude =18, side of square =15, diagonal of square = $15\sqrt{2}, r = \frac{15}{2}\sqrt{2}$,

$V = \frac{1}{3}Bh = \frac{1}{3} \bullet \left(\frac{15\sqrt{2}}{2}\right)^2 \bullet 18 \bullet \pi = 675\pi$

16. A Draw altitude from Q to $\overline{RP}$. Since the altitude is opposite a 45 degree angle, alt = $\frac{4}{\sqrt{2}} = 2\sqrt{2}. A = \frac{1}{2} \bullet 2\sqrt{2} \bullet 10 = 10\sqrt{2}$

17. E Since chord is parallel to tangent, radius is perpendicular to tangent and the chord. Let part of radius each be equal to x, radius = 2x. Draw radius forming a right triangle, use Pythagorean Theorem $x^2 + 36 = (2x)^2$, $x = 2\sqrt{3}, C = 8\pi\sqrt{3}$

18. D Drawn 15 inch segment from center of sphere to center of circle formed. Then this segment is perpendicular to and bisects chord. Draw radius of sphere and use Pythagorean Theorem to find the length of the radius – which is 25, diameter is 50.

19. A
$d = \sqrt{l^2 + w^2 + h^2}, 5\sqrt{2} = \sqrt{16 + 9 + w^2},$
$w = 5, V = 4 \bullet 3 \bullet 5$ which is 60.

20. E Let sides be 8 and 6 making area 48. New length = 12 so new width must be 4. Decrease of $33\frac{1}{3}\%$.

21. C Angle bisector causes side to be proportional to parts of the 3rd side. $\frac{12}{x} = \frac{9}{6}, x = 8, p = 35.$

22. A Radius is 2 and height is 10.
$LA = 2\pi rh = 40\pi$

23. C Tangents from same external point are congruent so ER=EU and FR=FS, since ER=RF, all 4 of these segments are congruent and their length is 19. UH=HT=TG=SG=2(22), p=19(4)+2(22)=120.

24. D
$2B + ph = 592, 2(12w) + (24 + 2w)8 = 592, w = 10$

25. D $\frac{4}{3}\pi \bullet 27 - \frac{4}{3}\pi \bullet 8 = \frac{76}{3}\pi$

26. C $V = \pi^2 h = \pi(2r)^2 h = 4\pi^2 h$

27. C Radius from center to point of tangency must be perpendicular to 2x+y=5, slope of radius is $\frac{1}{2}$ and passes through (0,0), Equation of radius is $x - 2y = 0$. Find the point of intersection of radius and tangent line – (2,1). Distance from (2,1) to (0,0) is $\sqrt{5}$ which is the radius. C= $2\pi\sqrt{5}$.

28. A Label triangle ABC with B the right angle. Draw radius to both legs forming a square. Let P be the vertex of the square on BC and Q on BA. Let R be the point of tangency of the hypotenuse. Let RC=x, AR=20-x, AQ=20-x,PC=x,BP=BQ=4. Use Pythagorean Theorem and find values of x are 8 or 12. Both values give sides of 12 and 16. So p=48.

29. C $\frac{\frac{4}{3}\pi r^3}{\pi r^2 \bullet 2r} = \frac{2}{3}$

30. B Perimeters have ratio of 15:8, areas would be 225:64. $\frac{225}{64} = \frac{100}{x}, x = 28.4$