Mu Alpha Theta National Convention 2003 Calculus - The Number "e"

If none of the answers given is correct choose e) NOTA.

1. Find the slope of the normal to $y = e^{\sin(2x)}$ at $x = \frac{\pi}{12}$.

a) $\frac{-\sqrt{3e}}{3e}$ b) $\frac{-2\sqrt{3e}}{3e}$
c) $\frac{2\sqrt{e}}{e}$ d) $\sqrt{3e}$ e) NOTA

2. The first person to use e to symbolize the base of the natural logarithms was

a) Briggs b) Descartes
c) Euler d) Napier e) NOTA

3. $\int e^x(1+\sec(e^x))dx =$

a) $e^x + \ln|\sec(e^x)+\tan(e^x)| + C$

b) $x + \ln|\sec(e^x)+\tan(e^x)| + C$
c) $e^x + \sec(e^x)\tan(e^x) + C$
d) $x + \ln|\sec(e^x)\tan(e^x)| + C$
e) NOTA

4. For a population of female African elephants the weight W(t) in kilograms at age t (in years) may be approximated by the von Bertanlanffy growth function

$$W(t) = 2600(1 - 0.51e^{-0.075t})^3.$$

If an adult female weighs 1800 kg at the present, use the equation to approximate A, her current age to the nearest year. Find the tenths digit of ln(A).

a) 4 b) 6 c) 7 d) 9 e) NOTA

5. Who was the first person to prove that e was transcendental?

a) Noether b) Argand
c) Hermite d) Euler e) NOTA

6. Write the equation of the tangent line to $e^{xy} + 3y = 5$ at $x = 0$.

a) $5x + 9y = 15$ b) $4x - 9y = -12$
c) $y = \frac{5}{3}$ d) $4x + 9y = 12$ e) NOTA

7. If $\frac{dy}{dx} = 3e^{2x} + 6e^{-3x}$ and $y(0) = 4$, find y(1) to the nearest thousandth.

a) 10.984 b) 15.484
c) 15.683 d) 48.735 e) NOTA

8. A radioactive substance has a half-life of 5 days. How long in hours will it take for an amount to disintegrate to the extent that only 5% of the original amount remains? Give your answer to the nearest hour.

a) 22 b) 49 c) 486 d) 519 e) NOTA

9. Which of the following functions grows faster than e^x?

a) x^e b) $4e^x$
c) $500(2)^x$ d) $2^{(2x)}$ e) NOTA

10. $f(x) = \sin^{-1}(e^{3x+1})$. Find $f^{-1}(x)$.

a) $\frac{\sin(\ln x) - 1}{3}$ b) $\sin(\ln(3x + 1))$
c) $\frac{3e^{3x+1}}{\sqrt{1 - e^{6x+2}}}$ d) $\frac{\ln(\sin(x)) - 1}{3}$ e) NOTA

11. An error function in statistics is denoted by $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. Use the trapezoidal rule with n = 4 to approximate erf (2) to the nearest thousandth.

a) 0.994 b) 1.032
c) 3.281 d) 3.975 e) NOTA

12. A rectangle has two of its vertices on the x-axis and two on the curve $y = 2e^{-x^2}$. Find the perimeter of the rectangle of maximum area that can so be constructed.

a) $\frac{2\sqrt{2e}}{e}$ b) $\frac{\sqrt{2}\,e + 4\sqrt{e}}{e}$
c) $\frac{\sqrt{2}\,e^2}{4}$ d) $\frac{2\sqrt{e} + 2\sqrt{2}}{4}$ e) NOTA

13. Find the linearization at x = ln(3) of $f(x) = 4 + \int_{2\ln(3)}^{2x} (2e^{2t} + 3)dt$.

a) $42x - y = 42\ln(3) - 4$
b) $330x - y = 330\ln(3) - 4$
c) $21x - y = 21\ln(3) - 46$
d) $(21\ln(3))x - y = 21(\ln(3))^2 - 46$
e) NOTA

14. Which of the following is an example of a catenary curve?

a) $y = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$ b) $y = \frac{1}{\sqrt{e^{2x} + 1}}$
c) $y = \frac{e^x + e^{-x}}{2}$ d) $y = \frac{e^{x^2 - 3} + 1}{2}$ e) NOTA

15. Given that $f(x) = e^{\cos(x)}$ which of the following statements are true over $[0, 4\pi]$.

i) f has 3 critical values over $(0, 4\pi)$.
ii) f is concave up at $x = \frac{\pi}{2}$.
iii) $f'(\pi) = e^{-1}$.
iv) f increases over $(\pi, 2\pi)$ and $(3\pi, 4\pi)$.
v) The range of f is $[e^{-1}, e]$.

a) i, iii, iv, v b) ii, iii, iv, v
c) i, ii, iv, v d) i, ii, iv e) NOTA

16. Who discovered the relationship $e^{\pi\sqrt{-1}} + 1 = 0$?

a) Euler b) Taylor
c) Legrende d) Briggs e) NOTA

17. The mass m(t) in grams of a tumor t weeks after it begins growing is given by $m(t) = \frac{e^t}{10}$. To the nearest thousandth what is the average rate of change in grams per week during the third week of growth.

a) 0.127 b) 0.345
c) 1.270 d) 3.451 e) NOTA

18. $\int \frac{3-e^x}{e^{3x}}\,dx$ =

a) $\frac{2-e^x}{6e^{3x}}+C$ b) $\frac{e^x-2}{2e^{3x}}+C$
c) $\frac{3e^x-2}{2e^{3x}}+C$ d) $\frac{2-3e^x}{6e^{3x}}+C$ e) NOTA

19. $f(x) = 2e^{-2x} - 2$. Which of the following statements are true?

a) $f(0) < f'(0) < f''(0)$
b) $f''(0) < f(0) < f'(0)$
c) $f'(0) < f''(0) < f(0)$
d) $f'(0) < f(0) < f''(0)$
e) NOTA

20. This mathematician is famous for his personal notebooks of equations and his collaboration with Godfy Hardy. One of his most famous equations contains e, π, and the golden mean.

a) Riemann b) Skewers
c) Ramanujan d) Wiley e) NOTA

21. The Ebbinghaus Model for human memory is $P(t) = (100 - a)e^{-bt} + a$ where P(t) is the percentage retained after t weeks. If $a = 20$ and $b = 0.5$ at what rate to the nearest thousandth in percentage per week is the information being forgotten after 3 weeks. (The constants a and b vary from one person to another.)

a) 8.925 b) 17.850
c) 82.150 d) 91.075 e) NOTA

22. $\lim_{n\to\infty} \frac{e^{\frac{1}{n}} + e^{\frac{2}{n}} + e^{\frac{3}{n}} + \cdots e^{\frac{n}{n}}}{n}$ =

a) e - 1 b) $\frac{1}{e}$ c) e d) 0 e) NOTA

23. The line $y = -4x - 7$ is tangent to the graph of $y = e^{x^2-4}$ at the point (a, b). Find a + b.

a) -4 b) -1 c) e^{-4} d) 2 + e e) NOTA

24. Using Newton's Law of Cooling determine the reading on a thermometer to the nearest thousandth 5 minutes after it is taken from a room at 72°F to the outdoors where the temperature is 20°F , if the reading dropped to 48°F after 1 minute.

a) 18.215 b) 20.143
c) 22.354 d) 24.153 e) NOTA

25. According to the famous hat problem a hat-check girl has checked n hats, but they have become hopelessly scrambled. She hands back the hats at random. As n increases without bound what is the probability that no man gets his own hat?

a) $\frac{1}{e^2}$ b) $\frac{1}{e-1}$ c) $\frac{1}{2e}$ d) $\frac{1}{e}$ d) NOTA

26. Find a positive value of k such that when the region bounded by $y = e^{2x}$, $x = 0$, $y = 0$ and $x = k$ is rotated about the x-axis, the resultant solid has a volume of 8π.

a) $1.25\ln(2)$ b) $\ln\sqrt[4]{3}$
c) $0.5\ln(32)$ d) $0.25\ln(33)$ e) NOTA

27. The value of a yacht in dollars after t years of use is $V(t) = 250{,}000\ e^{-0.125t}$. To the nearest dollar what is the average value of the yacht over its first 10 years of use?

a) 126,370
b) 142,699
c) 156,321
d) 24, 998
e) NOTA

28. Find $\int_0^{\ln(\sqrt{2})} \frac{1 + \cos(e^{-2x})}{e^{2x}}\,dx$ to the nearest thousandth.

a) 0.431 b) 0.881
c) 1.732 d) 1.800 e) NOTA

29. $\lim_{x\to 0} \frac{ae^x - bx - 2}{x^2} = c$, where c is a real number. Find c.

a) 0 b) 0.5 c) 1 d) 2 e) NOTA

30. $P(t) = \frac{80}{8 - 4e^{\frac{-t}{5}}}$. The population of foxes in a forest is modeled by P, where P(t) is the number of foxes present in the forest in t weeks after an initial observation at $t = 0$. Which of the following statements is false?

a) As t increases the rate of change in the number of foxes tends to 0.
b) The tangent lines to the graph of P lie above the graph of the curve.
c) The initially observed population had 20 foxes.
d) To the nearest tenth the average rate of change in the population over the first 5 weeks was -1.5.
e) NOTA

The number "e" Mu Alpha Theta Nationals 2003

a 1. $y' = e^{\sin(2x)}\cos(2x)2$ $m_{tan} = e^{1/2}\cdot 2\left(\frac{\sqrt{3}}{2}\right)$ $m_{norm} = \frac{-\sqrt{3}e}{3e}$
$\sqrt{3e}$

c 2. Euler

a 3. $\int (e^x + e^x \sec(e^x))dx = e^x + \ln|\sec(e^x) + \tan(e^x)| + C$

d 4. $1800 = 2600(1 - 0.51e^{-.075t})^3$ $\frac{9}{13} = (1 - .51e^{-.075t})^3$ $\sqrt[3]{\frac{9}{13}} = |-.51e^{-.075t}$
$\sqrt[3]{\frac{9}{13}} - | = -.51e^{-.075t}$ $-.11536038 = -.51e^{-.075t}$ $.2261968 = e^{-.075t}$
$\ln(.2261968) = -.075t$ $t = 20$ $\ln(20) = 2.9957$

c 5. Hermite

d 6. $e^{xy}[xy' + y] + 3y' = 0$ $(0, 4/3)$ $1[0 + \frac{4}{3}] + 3y' = 0$ $3y' = -4/3$ $y' = -\frac{4}{9}$
$4x + 9y = 12$

b 7. $y = \frac{3}{2}e^{2x} - 2e^{-3x} + C$ (15.484)
$4 = \frac{3}{2} - 2 + C$ $C = \frac{9}{2}$ $y = \frac{3}{2}e^{2x} - 2e^{-3x} + \frac{9}{2}$ $\frac{3}{2}e^{2} - 2e^{-3} + \frac{9}{2}$

d 8. $\frac{1}{2} = e^{120R}$ $\frac{\ln(.5)}{120} = R$ $.05 = e^{\frac{\ln(.5)}{120}t}$ $\frac{120\ln(.05)}{\ln(.5)} = t$ (519)

d 9. $x = \sin^{-1}(e^{3y+1})$ $\sin(x) = e^{3y+1}$ $\frac{\ln(\sin(x)) - 1}{3} = y$

d 10. $\ln(\sin(x)) = 3y + 1$

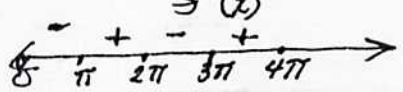
a 11. $\frac{2}{\sqrt{\pi}}\left(\frac{2}{8}\right)[e^0 + 2e^{-.25} + 2e^{-1} + 2e^{-2.25} + e^{-4}] = .994$

e 12. $A(x) = (2x)(2e^{-x^2}) = 4xe^{-x^2}$
$A'(x) = 4x[-2xe^{-x^2}] + 4e^{-x^2} = 4e^{-x^2}[-2x^2 + 1]$ $x = \frac{1}{\sqrt{2}}$
$L = \frac{2}{\sqrt{2}}$ $W = 2e^{-1/2}$ $P = \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{e}} = 2\sqrt{2} + \frac{4\sqrt{e}}{e} = \frac{2e\sqrt{2} + 4\sqrt{e}}{e}$

b 13. $S'(x) = 2[2e^{4x} + 3]$ $S'(\ln 3) = 2[2e^{4\ln 3} + 3] = 2[162 + 3] = 330$
$S(\ln 3) = 4$ $330x - y = 330\ln 3 - 4$

c 14.

c 15. $S'(x) = e^{\cos x} \cdot -\sin x$ $S''(x)e^{\cos x}(-\cos x) + (-\sin x)(-\sin x)e^{\cos x}$
i (T) $\sin x = 0$ $0, \pi, 2\pi, 3\pi, 4\pi$
ii (T) $S''(\frac{\pi}{2}) = 0 + (-1)(-1)(1) = 1$
iii (F) $S'(\pi) = 0$



iv (T)
v (T) $-1 \le \cos(x) \le 1$ $\therefore e^{-1} \le y \le e$

(a) 16. Euler

(c) 17. $\frac{m(3)-m(2)}{3-2} = \frac{e^3}{10} - \frac{e^2}{10} = 1.270$

(b) 18. $\int (3e^{-3x} - e^{-2x})\,dx = -e^{-3x} + \frac{e^{-2x}}{2} + C$ $= \frac{-1}{e^{3x}} + \frac{1}{2e^{2x}} + C$

$\frac{-2+e^x}{2e^{3x}} + C$

(d) 19. $S(0)=0$ $S'(x) = -4e^{-2x}$ $S'(0) = -4$ $S''(x) = 8e^{-2x}$ $S''(0) = 8$

$S'(0) < S(0) < S''(0)$

(c) 20. Ramanujan

(d) 21. $P'(3) = 8.925$ percentage retained percentage forgotten 91.075

(a) 22. $\int_0^1 e^x\,dx = |_0^1 e^x = e - 1$

(b) 23. $2xe^{x^2-4} = -4$ $xe^{x^2-4} = -2$ $x = -2$ $(-2, 1)$ -1

(c) 24. $\ln|T-20| = kt + C$ $t=0$ $T=72$ $\ln 52 = C$ $t=1$ $T=48$ $\ln|T-20| = kt + \ln 52$ $\ln 28 = k + \ln 52$ $k = \ln(28) - \ln(52)$

$\ln|T-20| = (\ln 28 - \ln 52)5 + \ln 52$

$\ln|T-20| = -.8560476$

$T = e^{-.8560470} + 20$

(d) 25. $\frac{1}{e}$

(d) 26. $\pi\int_0^K (e^{2x})^2\,dx = 8\pi$ $u = 4x$ $du = 4\,dx$

$\int_0^K e^{4x}\,dx = 8$ $\frac{1}{4}\int_0^{4K} e^u\,du = 8$ $|_0^{4K} e^u = 32$ $e^{4K} - e^0 = 32$ $e^{4K} = 33$ $K = \frac{\ln 33}{4}$

(b) 27. $\frac{1}{10}\int_0^{10} 250{,}000\,e^{-.125t}\,dt$ $25{,}000\int_0^{10} e^{-.125t}\,dt$ $u = -.125t$ $du = -.125\,dt$

$-200000\int_0^{-1.25} e^u\,du = -200{,}000\left[e^{-1.25} - e^0\right]$

(a) 28. $\int_0^{\ln\sqrt{2}-2} e^{-2x}\,dx + \int_0^{\ln\sqrt{2}} \frac{\cos(e^{-2x})}{e^{2x}}\,dx$ $u = e^{-2x}$ $du = -2e^{-2x}\,dx$

$u = -2x$ $du = -2\,dx$

$-\frac{1}{2}\int_0^{-\ln 2} e^u\,du$ $-\frac{1}{2}\int_1^{1/2} \cos u\,du = -\frac{1}{2}\left[(\sin(.5) - \sin(1)\right]$

$-\frac{1}{2}|_0^{\ln 1/2} e^u = -\frac{1}{2}(\frac{1}{2} - 1) = \frac{1}{4}$

$.1810227$

$+.125$

$.1431$

(c) 29. $\lim_{x\to 0}(ae^x - bx - 2) = 0$ $\therefore a - 2 = 0$ $a = 2$

$\lim_{x\to 0} \frac{ae^x - b}{2x}$ $\lim_{x\to 0} \frac{ae^x}{2} = c$ $\frac{a}{2} = c$ $c = 1$

(b) 30. a(T) $\lim_{t\to\infty} P(t) = 10$ b(F) graph is concave up c(T) $t=0, P=20$

d(T) $\frac{P(5) - P(0)}{5} = -1.549$