MATRICES AND DETERMINANTS – THETA
Mu Alpha Theta National Convention 2003

Note: For this test all uppercase variables on this test are matrices, all lowercase variables are real numbers, the notation A^Q stands for Transpose of A, I stands for the Identity Matrix, the notation B^{-1} refers to the inverse of B, and NOTA refers to None of These Answers is correct.

1) Find the area of a triangle with vertices of (4, 3), (-5, 2), and (-12, -10).

A) 50 B) 50.5 C) 101 D) 101.5 E) NOTA

2) If AB = I and $A = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$, then find the sum of the components for a solution for B that will make the above equation a true statement.

A) -2 B) 0 C) 1 D) 2 E) NOTA

3) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$, Find $(A^Q)(B^{-1})(B^Q)(A^{-1})$.

A) $\frac{1}{4}\begin{bmatrix} 33 & -7 \\ 13 & 5 \end{bmatrix}$ B) $\frac{1}{4}\begin{bmatrix} 31 & -15 \\ 30 & -14 \end{bmatrix}$ C) $\frac{1}{4}\begin{bmatrix} 27 & 13 \\ 32 & -18 \end{bmatrix}$ D) $\frac{1}{4}\begin{bmatrix} 32 & -15 \\ -14 & 31 \end{bmatrix}$ E) NOTA

4) For this problem the lower case letters in $(Mu\ Alpha\ Theta)$ correspond to the alphabet, where all five vowels (a, e, i, o, u) each have a value of 2 and each of the other 21 letters have a value equal to (log 1). Let $M = [2003],\ A = [2004],\ T = [2005]$.
Find the exact value of $Mu + Alpha + Theta$.

A) $[6{,}012]$ B) $[12{,}024]$ C) $[14{,}301]$ D) $[16{,}034]$ E) NOTA

5) Find the exact value of t. $\begin{cases} 7m + 2a + 5t = 1 \\ 2m + 2a + 7t = 2 \\ 8m + 1a + 9t = 3 \end{cases}$

A) $\frac{34}{83}$ B) $\frac{-2}{83}$ C) $\frac{-3}{83}$ D) $\frac{-33}{83}$ E) NOTA

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6) Evaluate: $\begin{vmatrix} 2 & 0 & 0 & 3 \\ 2 & 0 & 0 & 4 \\ 2 & 0 & 0 & 5 \\ 1 & 9 & 8 & 4 \end{vmatrix}$.

A) -2 B) -1 C) 1 D) 2 E) NOTA

7) Find the transpose for this matrix: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

A) $\begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$ C) $\begin{bmatrix} 3 & 2 & 1 \\ 6 & 5 & 4 \end{bmatrix}$ D) $\begin{bmatrix} 6 & 3 \\ 5 & 2 \\ 4 & 1 \end{bmatrix}$ E) NOTA

8) Let $A = \begin{bmatrix} 64 & 32 & 16 & 8 & 4 & \dots \\ 32 & 0 & 0 & 0 & 0 & \dots \\ 16 & 0 & 0 & 0 & \dots & \\ 8 & 0 & 0 & \dots & & \\ 4 & 0 & \dots & & & \\ \dots & \dots & & & & \end{bmatrix}$,

Find the sum of all the components in matrix A if the patterns continue to infinity.

A) 128 B) 184 C) 192 D) 256 E) NOTA

9) Let $\begin{bmatrix} 3y+2 & 5 \\ x-1 & 2 \end{bmatrix}\begin{bmatrix} -3 \\ 4y \end{bmatrix} = \begin{bmatrix} 16 \\ 22 \end{bmatrix}$, Find $x+y$.

A) -1 B) 0 C) 1 D) 2 E) NOTA

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10) Which of the following is equivalent to $\left(\left(\frac{1}{2}\right)^{10}\right)\left(\begin{bmatrix}1 & 3\\ 2 & 8\end{bmatrix}\begin{bmatrix}8 & -3\\ -2 & 1\end{bmatrix}\right)^{10}$?

A) $\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ B) $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ C) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ D) $\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix}$ E) NOTA

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11) Find the difference: $\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}-\begin{bmatrix}7 & 8\\ 9 & 10\end{bmatrix}$

A) $\begin{bmatrix}8 & 10 & 3\\ 13 & 14 & 10\end{bmatrix}$ B) $\begin{bmatrix}-6 & -6 & -3\\ -5 & -4 & -6\end{bmatrix}$ C) $\begin{bmatrix}-6 & -6 & 3\\ -5 & -4 & 6\end{bmatrix}$

D) Not Defined E) NOTA

12) Using Cramer's Rule, find $\frac{D_z}{D_y}$ for the following system: $\begin{cases}2x-4y+3z=3\\ 5x-z+2y=4\\ 4x-3y+2z=5\end{cases}$

A) -3 B) 0 C) 3 D) 6 E) NOTA

13) Find the sum of the components in the product of $\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}\begin{bmatrix}0\\ 2\\ 0\end{bmatrix}$.

A) 29 B) 30 C) 31 D) Not Defined E) NOTA

14) Let x, y, and z all be positive real numbers.

Find $x+y+z$, if $\begin{bmatrix}2x^2\\ y^3\\ 6z^2\end{bmatrix}=\begin{bmatrix}(5x+3)\\ (2x+2)\\ (7z+6x+y)\end{bmatrix}$.

A) 7.5 B) 8.5 C) 9.25 D) 10 E) NOTA

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15) Find the positive value of x if the following matrix has a determinant of -4. $\begin{bmatrix} x & 2 \\ 8 & x \end{bmatrix}$.

A) 0 B) 12 C) $2\sqrt{3}$ D) 4 E) NOTA

16) Which of the answers below for x would make the determinant of the following equal to zero? $\begin{vmatrix} 2 & 1 & 3 & -4 \\ -6 & 3 & -1 & -12 \\ 4 & -2 & 4 & 8 \\ 11 & -4 & 3 & x \end{vmatrix}$.

A) -4 B) 12 C) 16 D) 22 E) NOTA

17) Simplify $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^{2003}$.

A) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ E) NOTA

18) Which of the following is always equivalent to $(AB)^Q$?

A) $A^Q B$ B) $B^Q A^Q$ C) $A^Q B^Q$ D) BA^Q E) NOTA

19) Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & w & 0 \\ 0 & 0 & w^2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & 0 & 0 \end{bmatrix}$. If $BA = wAB$, $a \neq 0$.
then find the sum of non-negative real solutions for w.

A) none B) a C) 0 D) 1 E) NOTA

20) Which of the following matrices are skew?

A) $\begin{bmatrix} 0 & 8 \\ -8 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 & 8 \\ 8 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 8 & 0 \\ 8 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 0 & -8 \\ -8 & 0 \end{bmatrix}$ E) NOTA

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21) Which of the following matrices are singular?

A) $\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ D) $\begin{bmatrix} -1 & 0 \\ 1 & -1 \end{bmatrix}$ E) NOTA

22) Find the sum of the coefficients for the determinant of this matrix:
$\begin{bmatrix} (2x^2 - 4x + 6) & (4x^2 - 8x + 12) \\ (3x^2 - 6x + 9) & (5x^2 - 10x + 15) \end{bmatrix}$.

A) -4 B) 0 C) 2 D) -8 E) NOTA

23) Simplify: $\dfrac{\begin{vmatrix} 3 & 1 & 0 \\ 2 & x & 1 \\ 1 & 2 & 4 \end{vmatrix}}{\begin{vmatrix} x & 1 & 2 \\ 0 & 2 & 1 \\ 3 & 1 & 2 \end{vmatrix}}$. For values of x for which the expression is defined.

A) $\dfrac{12x+15}{5x+15}$ B) $\dfrac{12x-13}{5x+15}$ C) $\dfrac{12x+15}{5x-12}$ D) $\dfrac{12x-13}{3x-9}$ E) NOTA

24) Given that $A, B, \& C$ are all n X n matrices. Which of the following is not always true?

A) $AB = BA$ B) $B + C = C + B$ C) $C^{-1}I = IC^{-1}$ D) $A = A$ E) NOTA

25) Let the matrix equation AX = B and C be the inverse of A, to solve for X you must do which of the following products ?

A) BC B) CX C) XC D) CB E) NOTA

26) Let $\begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix}\begin{bmatrix} 2 & -3 \\ 4 & -7 \end{bmatrix}\begin{bmatrix} x \\ 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -x \end{bmatrix}$, Find the value of x.

A) 3 B) 4 C) 6 D) 12 E) NOTA

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27) Let $\begin{cases} 2x - y - 6z = 9 \\ x - 9y - 7 = 6 \\ -6x + 8y + 6z = -8 \end{cases}$, Find ab if

a = The sum of the elements for the coefficient matrix for the above system.
b = The sum of the elements for the augmented matrix for the above system.

A) 98 B) 60 C) -45 D) -32 E) NOTA

28) Find the greatest number for n that makes the following statement true.

$$\begin{bmatrix} n & 0 \\ 0 & y \end{bmatrix}^3 = \begin{bmatrix} -8 & 0 \\ 0 & 27 \end{bmatrix}^6$$

A) -2 B) 8 C) 64 D) 128 E) NOTA

29) Let $4^{\begin{vmatrix} x & 2 \\ 3 & 2x \end{vmatrix}} = \log_2 256$. The non-negative solution for x can be expressed in the simplified form of $\frac{\sqrt{a}}{b}$, where a and b are integers. Find ab.

A) 14 B) 30 C) 42 D) 60 E) NOTA

30) Let $A = \begin{bmatrix} 2 & -4 \\ -1 & -2 \\ 3 & -8 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 \\ 3 & -6 \end{bmatrix}, C = \begin{bmatrix} 2 & -4 \\ -4 & 8 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & -1 \end{bmatrix}$

Let $W = D(A(C + B))$

Find the sum of the elements for the simplified expression of W.

A) -21 B) -20 C) -19 D) -18 E) NOTA