- 1. A The easiest way to see this is to rewrite the numerator as x 3 + 1 allowing us to rewrite the latter function as  $y = 1 + \frac{1}{x-3}$  at which point the two translations should be obvious (the same form of the equation can be obtained using polynomial division.
- 2. D  $\sin(2x) = 2\sin(x)\cos(x)$ ; once this substitution is made and other trig functions are written in terms of sin and cos, the expression becomes  $\sin^2 x + \cos^2 x$  which equals 1.
- 3. D This function has a vertical asymptote and a slant asymptote. The vertical one is easy, where the function is undefined, x=1. The slant asymptote may be obtained by polynomial division, which yields y = 5x + 2 after discarding the remainder, making the point of intersection (1,7)
- 4. B Since all the choices have a slope of -2 one can simply solve for the point of intersection between  $y = \frac{1}{2}x$  and the unit circle and check if that point is on the line since the radius must be perpendicular to the tangent line at the point of tangency. Solving that system yields the point  $(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5})$  which is only on line II.
- 5. B Rewriting the first equation by squaring both sides and completing squares reveals it is a circle of radius 4 centered at (2,0). This means the solid produced is a cylinder of radius 6 and height 8 with a spherical hole of radius 4. Subtracting the two volumes yields  $\frac{608\pi}{3}$ .
- 6. A The locus of all points equidistance to (0,2) and (2,0) is their perpendicular bisector, which is y = x. For the point to also lie on y = 2x + 1, we have x = 2x + 1, or x = -1. So a + b = (-1) + (-1) = -2.
- 7. D A left sum would yields the upper estimate of 15 (10 + 5) and the right sum would yield the lower estimate of 7 (5 + 2).
- 8. D Only this function has a removable discontinuity at x=2 (the expression x-2 may be divided out), which would yield the behavior asked for.
- 9. D The function f(x) = x is a sufficient example. If n were even, the end behaviors would have to match. If a were negative then the limits would be reversed.
- 10. C The other two pieces of the piece-wise function terminate at (-2, -4) and (2, 4). The line y=2x passes through both points, creating continuity.
- 11. D This limit does not having matching one-sided limits. The limit from the left is 1 while the limit from the right is 2.
- 12. C These limits are often committed to memory, but can be found easily using L'Hopital's rule (a=1 and b=0).
- 13. B The presence of the expression  $\frac{dx}{dt}$  (which is not 0) and the absence of  $\frac{dh}{dt}$  should be sufficient to demonstrate this.
- 14. B Recognizing the double-angle identity for sine here (along with the recognition that cosine is an even function) saves considerable time. The expression then becomes  $\frac{d}{dx}$  [sin(10 2x<sup>2</sup>)] for which a chain rule yields  $-4x\cos(10 2x^2)$

- 15. These are limit definitions of the derivatives of tan(5x) and sec(5x) respectively. Therefore  $a = 5\sec^2(5x)$  and  $b = 5\sec(5x)\tan(5x)$ . Dividing and simplifying yields csc (5x)
- These conditions correspond to the graph being increasing and concave down. 16.
- The Mean Value theorem guarantees II. The graph of  $y = x^2 + 1$  provides an easy 17. counter example to the other two conditions.
- 18. A While f(x) has a local minimum at x=2, the value guaranteed by the Extreme Value Theorem occurs at x = -2, which has a corresponding y value of -8
- 19. The region forms two right isosceles triangles with legs of length 2 and 6.
- 20. E One way to do this is to add the second and fourth integral and then add the resulting integral to the third. This would result in the reverse of the integral asked for (a value of -1). Then one simply negates the result.
- Multiplying the integrand by  $\frac{6}{6}$  and using the u-substitution  $u = 3x^2 + 5$ ; du = 6x21. reveals the coefficient as  $\frac{1}{3}$  (the only distinction between the answers).
- 22. One may recognize a similarity in form to the derivative for arcsec(x). The correct function is  $arcsec(e^{-t}) + C$ .
- 23. The integrand in I is itself odd, therefore that statement is true. The problem with II is that the limits of integration should be reversed in the first integral (or the second) or one of the sides of the equation negated.
- 24. The integral produces the expression  $\frac{2k^3}{3}$ . Setting it equal to 18 and solving yields
- 25. If we split the first integral in I into two separate integrals the equation is a direct C application of integration rules for even and odd functions. If we apply the definition of an odd function on the right side of II and reverse the limits of integration while negating that side, we get a direct statement of the rule for the integration of even functions. The integrand, being a composition of functions in which at least one is even, is itself even.
- This function is in the 1<sup>st</sup> quadrant on the interval [0,3]. The average value is given 26. by the expression  $\frac{1}{3}\int_0^3 3x - x^2 dx$  which equals 1.5
- First, it should be noted that f(t) is a constant function. Therefore, the -4 doesn't 27. come into play at all. Next, if we rearrange the expression a bit, much of it simplifies. The first two terms in each integrand can be combined into a single integral from -3 to 3 which is equal to zero because the integrand is odd. The same can be done with the third term, which is even and thus simplified to  $2\int_0^3 x^2 dx$ which equals 18. Finally you have  $\int_{-3}^{1} 5x dx = -20$ . Adding these gives you -2 Let  $(g(t) = \sin(t^2))$ , then  $f(x) = G(3) - G(x^2)$  where G is the anti-derivative of
- 28. g.  $f'(x) = -2xg(x^2) = -2x\sin(x^4)$
- Since acceleration is the derivative of velocity,  $a(t) = \frac{e^{\sec^{-1}(t)}}{t\sqrt{t^2-1}}$  and  $a(2) = \frac{\sqrt{3}e^{\frac{\pi}{3}}}{6}$ 29.
- 30. This must be done implicitly. Let  $y = x^{\sin(x)}$ . After taking the natural log of both sides, rewriting the right hand side as a product, and differentiating with respect to x

we get  $\frac{1}{y}\frac{dy}{dx} = \frac{\sin(x)}{x} + \ln(x)\cos(x)$ . After substituting for y and solving for  $\frac{dy}{dx}$  we get  $x^{\sin(x)} \left[ \frac{\sin(x)}{x} + \ln(x)\cos(x) \right]$