NUMBER THEORY – ALPHA
Mu Alpha Theta National Convention 2003

For all questions, e. not given, means none of the above answers is correct.

1. If n is a positive integer, which of the following cannot be the units digit of 3^n ?

a. 1 b. 3 c. 5 d. 7 e. not given

2. For how many pairs of integers (a, b) where a < b does a multiplied by b equal 30?

a. 2 b. 3 c. 4 d. 5 e. not given

3. If a + k = a for all values of a, what is the value of k?

a. –a b. -1 c. a d. 0 e. not given

4. If a + b = 12 and both a and b are positive integers, what is the largest positive value for a?

a. 12 b. 11 c. 10 d. 9 e. not given

5.
$$\begin{array}{r} A2C8 \\ -\ A79 \\ \hline C66B \end{array}$$

If A, B and C are distinct digits in the correctly worked subtraction problem to the left, what is the sum of A, B and C?

a. 18 b. 16 c. 14 d. 12 e. not given

6. Coach Pratt lined his students up in a row. He said to Sally, "Pick the 12^{th} and 13^{th} kids in the line for they will be today's team captains". Sally replied, "Coach, from which end do I start"? "Sally", replied Coach, "It does not matter." How many kids were in line?

a. 26 b. 25 c. 24 d. 23 e. not given

7. Which of the following set(s) of numbers has (have) the property that the sum of any two numbers in the set is also in the set?

I. the set of even integers.
II. the set of prime numbers
III. the set of positive integers.

a. I only b. II only c. I and II only d. II and III only e. not given

8. When x = 10, the expression $\sqrt{1+2+3+x}$ has the value of 4. How many other values of x < 10 for which $\sqrt{1+2+3+x}$ has an integral value?

a. 6 b. 4 c. 3 d. 1 e. not given

9. If x and y are both positive, x is even and y is odd, which of the following must be odd?

a. xy b. x + 2y c. x^y d. y^x e. not given

10. The gcf of $24x^4y$ and $30x^3y^5$ can be expressed as ax^by^c, where a, b and c are positive integers. What is the sum of a + b + c?

a. 10 b. 19 c. 129 d. 136 e. not given

11. If 387,a42 is divisible by 3, then how many possible values of a are there?

a. 9 b. 5 c. 4 d. 3 e. not given

12. The product ab = 1200 and b is an odd number. Find a + b where b is the greatest possible integral value that satisfies the stated conditions.

a. 91 b. 243 c. 324 d. 1201 e. not given

13. What is the sum of all natural numbers less than 200 with exactly three factors and is not a multiple of 3?

a. 369 b. 368 c. 364 d. 360 e. not given

14. How many factors of 1800 are multiples of 10?

a. 12 b. 18 c. 24 d. 36 e. not given

15. In how many zeros will the product of (20)(25)(30)(35)(40)(45)(50) end?

a. 9 b. 8 c. 7 d. 6 e. not given

16. What is the sum of the four smallest, natural numbers that are each congruent to 5 mod 7?

a. 57 b. 62 c. 83 d. 90 e. not given

17. Find the number of positive integers that leave a remainder of 4 when divided into 4000.

a. 12 b. 18 c. 24 d. 30 e. not given

18. Find the largest possible, positive difference between any two numbers between 50 and 100 that have an odd number of factors?

a. 17 b. 23 c. 29 d. 35 e. not given

19. a, b, c and d are distinct numbers. Determine the total number of divisors for the number which is the LCM for $a^4b^3c^2d, a^7b^5c^3d$ and $a^5b^4c^3d^2$.

a. 120 b. 210 c. 576 d. 1080 e. not given

20. Given that n! ends in exactly 27 zeros, and n is a multiple of three, what is the largest possible value of n?

a. 120 b. 119 c. 118 d. 117 e. not given

21. The smallest natural number which is twice a perfect square is one-fifth of a perfect cube lies between

a. 225 and 300 b. 150 and 225 c. 75 and 150 d. 0 and 75
e. not given

22. Let the operation * be defined by a * b = the sum of all twin primes between a and b. Find 25 * 75.

a. 612 b. 512 c. 484 d. 408 e. not given

23. How many base 10 counting numbers will have a three digit representation in bases 4, 5 and 7?

a. 15 b. 75 c. 242 d. 334 e. not given

24. If the number A3640548981279644B is divided by 99, compute the value of A + B.

a. 6 b. 8 c. 10 d. 12 e. not given.

25. For how many positive integral values of n does n! end in precisely 25 zeros?

a. 6 b. 7 c. 8 d. 9 e. not given

26. How many 2-digit numbers, neither of whose digits is 0, are such that the product of their digits is a square?

a. 11 b. 13 c. 15 d. 17 e. not given

27. The integers a, b, and c are each greater than 20. One of them has an odd number of divisors; the other two each have three divisors. If a + b = c, compute the smallest possible value of c.

a. 169 b. 121 c. 49 d. 25 e. not given

28. The integers x, y and z are each perfect squares, and $x > y > z > 0$. If x, y and z form an arithmetic progression, compute the smallest possible value of x.

a. 121 b. 81 c. 49 e. 25 e. not given

29. The smallest positive integer with exactly 12 divisors (including 1 and the number itself) as divisors is 60. What is the sum of the next three, smallest natural numbers with exactly 12 divisors?

a. 312 b. 288 c. 264 d. 252 e. not given

30. Definition: If a, k and n are positive integers with $k > 1$, and $n = ka$, then a is called a "proper divisor" of n. How many positive integers less than 54 are equal to the product of their proper divisors?

a. 17 b. 16 c. 15 d. 14 e. not given