E. NOTA means "None Of These Answers is Correct." Figures may not be drawn to scale.

1. If units are disregarded, the lateral surface area of a cone with diameter 6 and height 4 is the same as the height of a cylinder. If the radius of the cylinder is $\dfrac{1}{\sqrt{\pi}}$, what is the volume of the cylinder?

A. 15π B. $12\pi(3+\sqrt{13})$ C. 24π D. $12\pi\sqrt{13}$ E. NOTA

2. Let A be the solid formed when an equilateral triangle is rotated around one of its sides. Let B be the solid formed when a triangle congruent to the first is rotated around one of its altitudes. What is the ratio of the volume of A to the volume of B?

A. $\dfrac{\sqrt{3}}{6}$ B. $\dfrac{\sqrt{3}}{3}$ C. $\sqrt{3}$ D. $2\sqrt{3}$ E. NOTA

3. If a regular polyhedron has x faces and y vertices, then the regular polyhedron that has y faces and x vertices is called a *dual polyhedron*. For example, the dual polyhedron of a tetrahedron is a tetrahedron, because it has four vertices and four faces, and the exchange results in a tetrahedron. What is the dual polyhedron of a regular octahedron?

A. Tetrahedron   B. Hexahedron   C. Octahedron   D. Dodecahedron   E. NOTA

4. What is the surface area of a sphere whose radius is 10?

A. 100π B. 400π C. $\dfrac{4000\pi}{3}$ D. 1600π E. NOTA

5. A pyramid with a square base is inscribed in a hemisphere of radius r such that the top of the pyramid lies on the center of the great circle and the base of the pyramid is parallel to the great circle. If the height of the pyramid is 3, what is the volume of the pyramid in terms of r?

A. $\dfrac{r^2}{2}$ B. $2r^2$ C. $2r^2-18$ D. 9   E. NOTA

6. The length of a right rectangular prism is increased by $33\dfrac{1}{3}$% and the width is decreased by 75%. If the volume is to remain unchanged, by how much is the height increased?

A. The height remains the same   B. 50%   C.100%   D. 300%   E. NOTA

7. In three dimensions, what is the set of all points for which $x=1$ and $y=2$?

A. A plane parallel to the xy-plane   B. A line parallel to the z-axis
C. A point at (1,2,0)   D. A plane parallel to the z-axis   E. NOTA

8. A roll of tape is formed by the 2 cm extension of the annulus between two concentric circles into a third dimension, where the radii of the circles are 5 cm and 6 cm. In terms of cm^3, how much tape is on the roll?

A. 2π B. 18π C. 60   D. 22π E. NOTA

9. A right circular cone has a vertex at the point (1,2,9) and a circular base centered at the point (1,2,0) which passes through the point (4,2,0). The plane $z = 5$ intersects the cone and forms an ellipse. If a sphere, S_1, that is inscribed in the region between the vertex and the ellipse, and another sphere, S_2, that is internally tangent to the cone below the ellipse and another sphere (called Dandelin spheres) are both tangent to the ellipse at its foci, what is the area of this ellipse? (Note: Figure is not drawn precisely)

(1,2,9)

z=5

(1,2,0)   (4,2,0)
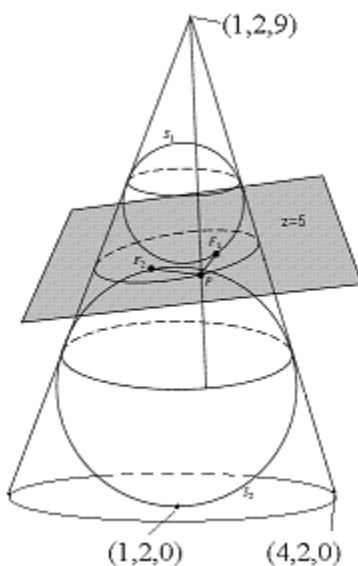
A. $\dfrac{8\pi}{3}$ B. $\dfrac{16\pi}{9}$ C. 25π D. Cannot be determined   E. NOTA

10. A two-dimensional arrow is rotated $360°$ around the axis of symmetry shown by the dotted line, which runs down the center of the arrow. What is the volume of the resulting figure?

2

6

2   10

11
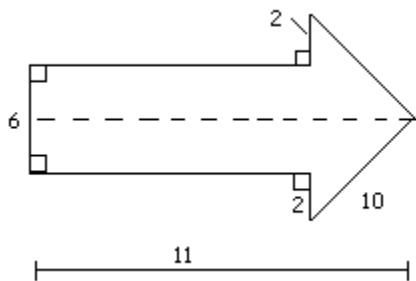
A. $66\pi + 5\pi\sqrt{3}$ B. $66\pi - 5\pi\sqrt{3}$ C. $99\pi + \dfrac{10\pi\sqrt{3}}{3}$ D. $99\pi - \dfrac{10\pi\sqrt{3}}{3}$ E. NOTA

11. The total surface area of a cone is equivalent to the total surface area of a cylinder. Their radii are the same and the height of the cone is three times that of the cylinder. If the height of the cone is k times the radius of the cone, then $k = $?

A. 3　　　　　　B. $\dfrac{12}{5}$　　　　　　C. $\dfrac{5}{4}$　　　　　　D. $\dfrac{1}{3}$　　　　　　E. NOTA

12. The space diagonal of a cube is $6\sqrt{6}$ in. What is the area of a face, in in^2 ?

A. 216　　　　　　B. 72　　　　　　C. $18\sqrt{2}$　　　　　　D. 36　　　　　　E. NOTA

13. A cubical box of length 8 inches will have an inner casing formed by connecting the centers of the faces of the cube. The centers of faces opposite one another are not connected. If the volume of the polyhedron formed by enclosing the casing is removed from the volume of the box, how many cubic inches remain?

A. $512 - 64\sqrt{3}$　　　　B. 256　　　　C. $\dfrac{1280}{3}$　　　　D. $\dfrac{1408}{3}$　　　　E. NOTA

14. The region bound by the lines x = 2, x = -2, y = 2, and y = 0 is rotated $360°$ around the line x = 2. What is the total surface area of the resulting figure?

A. 48π　　　　　　B. 32π　　　　　　C. 24π　　　　　　D. 16π　　　　　　E. NOTA

15. In the figure, DO = 2, BO = 8, AO = a, and OC = b. If $c = 3a$, $d = 2b$, and $c + d = ra$, what is the circumference of the great circle of a sphere whose radius is r?
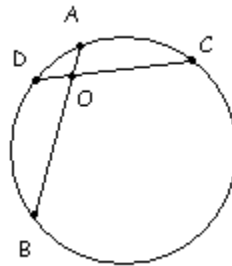
A. $\dfrac{25\pi}{8}$　　　　　　B. $\dfrac{25\pi}{4}$　　　　　　C. 11π　　　　　　D. 22π　　　　　　E. NOTA

16. A box in the shape of a right rectangular prism has two faces each with an area of $\sqrt{35}$, two faces each with an area of $7\sqrt{5}$, and a volume of 35. What is the sum of the areas of the other two faces?

A. $7\sqrt{5}$　　　　　　B. $5\sqrt{7}$　　　　　　C. $\sqrt{35}$　　　　　　D. $\dfrac{\sqrt{7}}{7}$　　　　　　E. NOTA

17. A right circular ice cream cone has a height of 8 in. and a radius of 2 in. Each scoop of ice cream is a sphere of radius 1 in. If the scoops are squished into the cone so that there is no empty space inside the cone and no ice cream protrudes over the top, how many scoops go into the cone? Assume the ice cream maintains a constant density.

A. 2   B. 4   C. 8   D. 16   E. NOTA

18. Two towers stand 76 feet apart. Each tower is a rectangular prism with a square base of area 484 sq. ft. If the taller tower is 91.5 feet tall, find the shortest distance from one corner at the top of the taller tower to the farthest opposite corner of the smaller tower, i.e. the distance from A to B. Note that both towers are perpendicular to the ground and the corresponding faces are parallel. (Hint: You can avoid time-wasting arithmetic if you know your Pythagorean triples.)
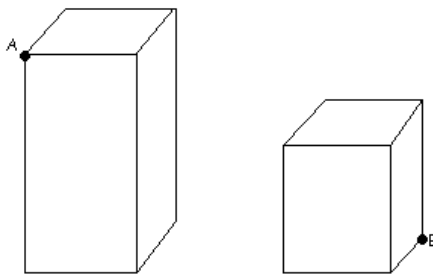
A. 121.0   B. 150.9   C.152.5   D. 233.5   E. NOTA

19. The sum of the volumes of a regular hexagonal pyramid and a regular hexagonal prism is 147. If the bases have the same area and the height of the pyramid is 4 times that of the prism, what is the volume of the pyramid?

A. 21   B. 63   C. 84   D. 126   E. NOTA

20. A cube with side length 8 inches is made up entirely of smaller white cubes with side length 1 inch. Suppose I paint the large cube blue on two opposing faces, red on two other opposing faces, and green on the last two opposing faces. What is the ratio, in most simplified form, of colored cubes (i.e. cubes which are not entirely white) NOT containing blue to cubes which are entirely white?

A. 2:3   B. 7:9   C. 32:27   D. 37:27   E. NOTA

21. A right prism has a convex pentagonal base formed by the points (-1,5,0), (6,7,0), (-1,7,0), (5,2,0), and the origin. The other vertices are located at (-1,5,9), (6,7,9), (-1,7,9), (5,2,9), and (0,0,9). What is the volume of the prism?

A. $\dfrac{225}{6}$ B. $\dfrac{225}{2}$ C. 225   D. 333   E. NOTA

22. The graph of $(x-3)^2 + y^2 = 9$ is rotated about the x-axis **180°**. What is the shape of the resulting figure?

A. Sphere   B. Hemisphere   C. Cone   D. Cylinder   E.NOTA

23. A sheet of paper is 100 microns thick and a micron is $\frac{1}{10000}$ cm. If the book contains 1000 pages (500 sheets of paper), what is the thickness of the book (not including the cover), in cm.?

A. 0.5   B. 1   C. 5   D. 10   E. NOTA

24. A cylinder with height 10 and radius 3 has volume 80π because the cylinder has a depression in the bottom that is part of a sphere. If the volume of this sphere is 45π, what fraction of the volume of the entire sphere is the part that forms the depression?

A. $\frac{1}{4}$ B. $\frac{1}{5}$ C. $\frac{1}{8}$ D. $\frac{1}{9}$ E. NOTA

25. What is the shortest distance between the point (1,4,1) and the plane $3x - 4y - z = 12$?

A. $\frac{7}{6}$ B. $\sqrt{26}$ C. $\frac{7\sqrt{2}}{3}$ D. $\frac{13\sqrt{2}}{3}$ E. NOTA

26. The shape of a soccer ball is a truncated icosahedron, formed with 12 regular pentagons and 20 regular hexagons. Which of the following also describes this figure?

A. Archimedean solid   B. Johnson solid   C. Platonic solid   D. Polyhedral compound
E. NOTA

27. A right circular cylinder with radius x and height z (x > 0, z > 0) has a spatial center (i.e. the center of the circle which intersects the cylinder at its midpoint) at the origin. How many of the following points are <u>necessarily</u> within or on the cylinder?

    I. (x, z, 0)
    II. (x/2, 0, z/2)
    III. (0, 0, z - x)
    IV. (-x, 0, x – z)

A. 0   B. 1   C. 2   D. 3   E. NOTA

28. If a woodchuck could chuck 800π ft^3 of wood, what is the maximum height, in feet, of a cylindrical tree with an 8 foot diameter he could chuck?

A. 16   B. 40   C. 50   D. 200   E. NOTA

29. A rectangular prism with a square base is inside a cylinder, as shown. The faces of the prism are on the same plane as the faces of the cylinder, and their vertices lie on the circles of the cylinder's bases. The cylinder has a radius of 1 and a surface area of 10π. What is the surface area of the inscribed prism?
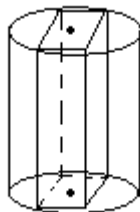
A. 8   B. $40\sqrt{2}+4$ C. 20   D. $4+16\sqrt{2}$ E. NOTA

30. A right circular cone at time $t=0$ has a slant height of 5. The measure of the vertex angle (the top angle of the triangular cross-section) is determined by the function $A(t)=t^{2}+7t+60$, with $A(t)$ in degrees. If the slant height remains constant, what is the difference in the areas of the cone's base at $t=5$ and $t=3$?

A. $\dfrac{125\pi(3-2\sqrt{2})}{24}$ B. $\dfrac{25\pi}{4}$ C. $\dfrac{3}{2}$ D. $\dfrac{125\pi(3-\sqrt{3})}{24}$ E. NOTA