2009 Alpha Individual Test

E IS NONE OF THESE

1. If $f(x)$ is a linear function and $f(2) = 3, f(-3) = 5$, find the value of $f(4)$.

   a) $\frac{11}{5}$ b) 2 c) $-\frac{11}{5}$ d) $-\frac{1}{2}$

2. Find the inverse of $f(x) = 2x^2 - 5x + 1$ for $x \geq \frac{5}{4}$.

   a) $y = \frac{15+\sqrt{8x+23}}{4}, x \geq -\frac{17}{8}$ b) $y = \frac{-5+\sqrt{8x+17}}{4}, x \geq -\frac{17}{8}$
   c) $y = \frac{5+\sqrt{16x+34}}{4}, x \geq -\frac{17}{8}$ d $y = \frac{5+\sqrt{8x+17}}{4}, x \geq -\frac{17}{8}$

3. $\sin x + \cos x = \frac{7}{5}$, find the value of $\cos 2x$.

   a) $\frac{24}{25}$ b) $\frac{7}{24}$ c) $\frac{7}{25}$ d) $\frac{5}{24}$

4. Simplify: $\log_{\sqrt[4]{3}} \log_{\sqrt[3]{10}} \log_2 1024$.

   a) $\sqrt{3}$ b) 4 c) $\sqrt[12]{10}$ d) $\sqrt[4]{3}$

5. Solve for X if $\begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix} X = \begin{bmatrix} -5 & -2 \\ 39 & 52 \end{bmatrix}$.

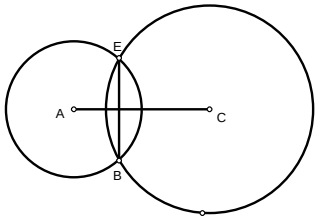
   a) $\begin{bmatrix} -1 & 3 \\ 7 & 8 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 3 \\ 7 & -8 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 3 \\ 7 & 8 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 3 \\ 7 & -8 \end{bmatrix}$

6. If $a^2 + b^2 = c^2$, where a, b,c are integers and $P = abc$ which of the following always divides P? (i.e. no remainder)

   a) 24 b) 36 c) 45 d) 60

7. Two night watchmen, Jackson and Smithson, arrange for an evening together away from work. Jackson is off duty every eighth evening starting today and Smithson is off duty every fifth evening starting tomorrow. In how many days from today can they get together?

   a) 16 b) 17 c) 33 d) 57

8. 

   The distance between the centers of the 2 circles is 14, and the radii of circle A and C are 13 and 15 respectively. Find the length of EB.

   a) 12 b) 24 c) 36 d) 48

9. Find the sum of the three terms: $\frac{1}{\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{4}} + \frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}} + \frac{1}{\sqrt[3]{9}+\sqrt[3]{12}+\sqrt[3]{16}}$.

a) $\sqrt[3]{36} + 2\sqrt[3]{18} + 4\sqrt[3]{9}$ b) $2\sqrt[3]{2} - 2\sqrt[3]{6}$ c) $\sqrt[3]{4} - 1$ d) $\sqrt[3]{4} + \sqrt[3]{9} + \sqrt[3]{12}$

10. $f(n) = 2, 5, 10, 17, \ldots, n \geq 1$, find $f(100)$.

a) 100007 b) 1000001 c) 10003 d) 10001

11. Given: $3^{x^2-2xy} = 1$ and $2log_3 x = log_3(y + 3)$, find the sum of all the y – values in the solutions.

a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $\frac{5}{4}$ d) $-\frac{1}{4}$

12. The real roots of $2x^3 - 7x^2 + kx - 2 = 0$ are in geometric progression. Find the r (common ratio).

a) 4 b) 2 c) ½ d) ¼

13. Evaluate: $cos\frac{5\pi}{4} - sin\,225^0 + tan\,750^0 - sec\left(-\frac{\pi}{4}\right)$.

a) $\frac{-\sqrt{3}-3\sqrt{2}}{3}$ b) $\frac{\sqrt{3}+3\sqrt{2}}{3}$ c) $\frac{-\sqrt{3}+3\sqrt{2}}{3}$ d) $\frac{\sqrt{3}-3\sqrt{2}}{3}$

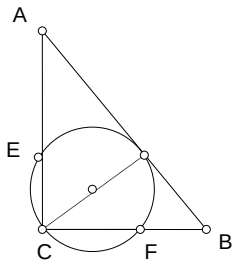
14. A local theatre needs to improve their net income and decides to raise the price of the night-time ticket price. Currently the night-time ticket costs \$6.00 and a study shows that for every \$0.25 rise in ticket price the attendance will drop 20 people. The average nightly attendance at this time is 1000 people. What should the theatre charge per ticket to maximize their income?

a) \$6.45 b) \$7.00 c) \$7.30 d) \$7.75

15. Referring to the previous question #14, what is the total increase in revenue with the new ticket price?

a) \$845 b) \$932 c) \$1256.50 d) \$1902

16. Altitude *CD* to hypotenuse *AB* of rt. Δ*ABC* is a diameter of circle *O*. This circle intersects *AC* in *E* and *BC* in *F*. If *AC* = 9 and *BC* = 12, find The length of *EF*.



a) 5 b) 6.5 c) 7.2 d) 9

17. Find the value(s) of x such that the matrix: $\begin{bmatrix} 1 & x & 0 \\ 2 & 3 & -1 \\ -1 & 6 & x \end{bmatrix}$ has no inverse.

a) -3, -1 b) 3, 1 c) -1, 3 d) 1, -3

18. Find $\sum_1^{30} \frac{4}{(n+3)(n+4)}$.

a) $\frac{45}{17}$ b) $\frac{15}{17}$ c) $\frac{34}{561}$ d) $\frac{2}{561}$

19. Express in rectangular form: $(2cis\ 30^\circ)^8$.

a) $128 - 128\sqrt{3}\,\boldsymbol{i}$ b) $-128 + 128\sqrt{3}\,\boldsymbol{i}$ c) $128 + 128\sqrt{3}\,\boldsymbol{i}$ d) $-128 - 128\sqrt{3}\,\boldsymbol{i}$

20. What is the solution for x in the following system of equations?

$$3x + 2y - z = 3$$
$$x + y + z = 2$$
$$2x + 3y + 2z = 3$$

a) -1 b) 1
c) -2 d) 2

21. $\frac{sin\ x}{1+cos\ x} + \frac{1+cos\ x}{sin\ x}$ may be simplified to yield:

a) 2 *cot x* b) 2 c) 2 *sec x* d) 2 *csc x*

22. Solve : $\frac{x-3}{2x+1} \geq 1$.

a) [-4, - ½] b) $(-\infty, -4] \cup (-\frac{1}{2}, \infty)$ c) [-4, - ½) d) $(-\infty, -4]$

23. A ladder leans against a building making a 75° angle with the ground. The end of the ladder reaches a point 30 ft. above the ground on the building. How long is the ladder?

a) $30(\sqrt{6} - \sqrt{2})$ b) $15(\sqrt{6} + \sqrt{2})$ c) $15(\sqrt{6} - \sqrt{2})$ d) $30(\sqrt{6} + \sqrt{2})$

24. If $P(x + 3) = x^2 + 7x + 4$ and $P(x) = ax^2 + bx + c$, find the ordered triple (a, b, c).

a) (1, 1, -8) b) (1, 13, 34) c) (-9, -21, 4) d) (1, -1, 4)

25. If $553_b + 670_b = 1003_b$, find the value of b.

a) 9 b) 12 c) 11 d) 7

26. The complex number $a + b\boldsymbol{i}$ when squared gives $-3 - 4\boldsymbol{i}$. If $a > 0$, find (a, b)

a) (1, 2) b) (2, -1) c) (1, -2) d) (1, -1)

27. If $f(x) = \frac{1-x}{1+x}$, find $f(f(x))$.

a) x b) 1 c) -1 d) $\frac{1-x^2}{1+x^2}$

28. Find the value of $\sqrt{3+\sqrt{8}} - \sqrt{3-\sqrt{8}}$.

a) 4 b) 6 c) 2 d) -2

29. Which of the following is a solution to: $cot(2\theta) - tan(\theta) = 1$ for $0 \leq \theta < 2\pi$.

a) $\frac{\pi}{4}$ b) $\frac{5\pi}{4}$ c) $\frac{9\pi}{4}$ d) $\frac{3\pi}{4}$

30. Find the area of the ellipse $4x^2 + 9y^2 - 4x - 3 = 0$.

a) 2π b) $2\sqrt{2}\,\pi$ c) 4π d) $\frac{1}{4}\pi$

Tie-Breakers:

1. Write the quadratic equation whose roots are the reciprocals of those of $5x^2 - 9x + 7 = 0$

2. Find $(1423)^2 - (1418)^2$.

3. Find the rational numbers x and y such that $(9 + 7\sqrt{2})(x + y\sqrt{2}) = 1$;