2009 Individual Theta Test

FOR EACH PROBLEM BELOW, NOTA MEANS "NONE OF THESE ANSWERS."

1. Evaluate: $(3^{-2} + 4^{-1})^{-2}$

    A. 97   B. $\dfrac{1296}{169}$ C. $\dfrac{169}{97}$ D. 169   E. NOTA

2. What is the remainder if $(3x^3 + 7x^2 + 5x + 7)$ is divided by $(x + 2)$?

    A. 1   B. 2   C. 0   D. 3   E. NOTA

3. Simplify: $log_{64}\left(\frac{1}{2}\right)$

    A. -1/6   B. 6   C. 1/6   D. -6   E. NOTA

4. Find the standard form of the equation of the parabola with focus (-1, -9) and vertex at (-1, -4).

    A. $y = -\frac{1}{20}(x + 1)^2 - 4$ B. $y = \frac{1}{20}(x + 1)^2 - 4$ E. NOTA

    C. $y = -\frac{1}{20}(x + 1)^2 - 9$ D. $y = -\frac{1}{20}(x + 1)^2 + 4$

5. What is the value of i^{2009}?

    A. 1   B. i C. $-i$ D. -1   E. NOTA

6. If $f(x) = 2x^2$ and $g(x) = x + 5$, find $g(f(3))$.

    A. 3   B. 128   C. 41   D. 23   E. NOTA

7. Evaluate $(1 + i)^8$.

    A. 16   B. -4   C. $8 - 8i$ D. $- 8i$ E. NOTA

8. In $\triangle ABC$, $\angle B = 30°$, $\angle C = 45°$, and $AC = 12$. What is the length of BC?

    A. $12\sqrt{12}$ B. $6\sqrt{2} + 6$ C. $6\sqrt{2} + 6\sqrt{6}$ D. $6\sqrt{6} + 12\sqrt{2}$ E. NOTA

9. If $f(x) = \frac{2}{3}x - \frac{1}{2}$, Then find $f^{-1}(x)$.

    A. $f^{-1}(x) = \frac{1}{2}x + \frac{2}{3}$ B. $f^{-1}(x) = \frac{3}{2}x + 2$ E. NOTA

    C. $f^{-1}(x) = \frac{3}{2}x + \frac{3}{4}$ D. $f^{-1}(x) = \frac{3}{2}x + \frac{1}{2}$

10. The width of a rectangle is increased by 60%. By what percent must its length be reduced in order for the area to remain the same?

    A. 60%   B. 37.5%   C. 40%   D. 62.5%   E. NOTA

11. Solve: $\sqrt{x-3} - x = -3$

    A. 3   B. 4   C. $\emptyset$ D. 3, 4   E. NOTA

12. The sides of a triangle measure 5 cm, 10 cm, and 13 cm. What is the area of the triangle in square centimeters?

    A. $6\sqrt{14}$ B. $12\sqrt{7}$ C. $5\sqrt{26}$ D. 65   E. NOTA

13. The roots of a polynomial with real coefficients are 2, 1 + 2**i**, and 1 − 2**i**. the equation of the polynomial is:

    A. $x^3 - 4x^2 + x + 6 = 0$ B. $x^3 + x + 10 = 0$ E. NOTA
    C. $x^3 + x - 10 = 0$ D. $x^3 - 4x^2 + 9x - 10 = 0$

14. IF $x + y = 4$ AND $x^3 + y^3 = 28$, Find the value of xy.

    A. 2   B. 3   C. 4   D. 7   E. NOTA

15. A rhombus with a 120° angle and a square both have sides measuring 10 inches. What is the ratio of the area of the rhombus to the area of the square?

    A. $1 : \sqrt{3}$ B. $\sqrt{3} : 2$ C. $\sqrt{2} : 2$ D. $1 : 1$ E. NOTA

16. Find the distance the point (5, 1) is from the line $3x + 4y = 4$

    A. 3.5   B. 3   C. 2   D. 2.5   E. NOTA

17. What is the sum of the units digits in 8^{2009}, 3^{2009}, and 7^{2009} ?

    A. 27   B. 22   C. 12   D. 18   E. NOTA

18. The segment joining $(5, 4)$ and $(1, 1)$ is rotated about the x-axis. What is the volume of the frustum formed?

    A. 84π B. 28π C. 20π D. 42π E. NOTA

19. In problem #18, at what point is the vertex of the cone located?

    A. $\left(-\frac{1}{3}, 0\right)$ B. $\left(0, -\frac{1}{3}\right)$ C. $\left(-\frac{1}{3}, -\frac{1}{3}\right)$ D. $\left(\frac{1}{3}, 0\right)$ E. NOTA

20. Simplify, assuming no division by zero: $\dfrac{\frac{x^2-9}{6x-12}}{\frac{x+3}{3x^2+3x-18}}$

    A. $\dfrac{(x+3)^2}{18(x^2-4)}$ B. $\dfrac{x^2-9}{3}$ C. $\dfrac{x^2-9}{18(x-2)^2}$ D. $\dfrac{x^2-9}{2}$ E. NOTA

21. Find the area of $16x^2 + 9y^2 - 96x + 36y + 36 = 0$.

    A. 48   B. 48π C. 12π D. 36π E. NOTA

22. Find the missing number in the sequence: $1, \dfrac{1}{32}, \underline{\quad}, \dfrac{1}{64}, \dfrac{1}{25}, \dfrac{1}{6}, 1$

    A. $\dfrac{1}{48}$ B. $\dfrac{1}{12}$ C. $\dfrac{1}{81}$ D. $\dfrac{1}{27}$ E. NOTA

23. Find the sum of the values of x that make $(x^2 + 6x + 6)^{x^2-7x+6} = 1$.

    A. 11   B. 13   C. 0   D. 1   E. NOTA

24. A cubic polynomial with integral coefficients has a root $3 - 4i$ and possesses the form
    $y = x^3 + ax + b$. Find $\dfrac{AB}{25}$.

    A. -66   B. 66   C. -17   D. 25   E. NOTA

25. What is the sum of the infinite geometric series $10 + 5 + 2.5 + 1.25 + \ldots$?

    A. 5   B. 20   C. 19   D. 12.5   E. NOTA

26. Given $A = \begin{bmatrix} 3 & 5 & -1 \\ 2 & 0 & -4 \\ -3 & 2 & -1 \end{bmatrix}$, Find $|A|$.

    A. 0   B. 22   C. 84   D. 90   E. NOTA

27. Simplify: $\sqrt[5]{(9^{2.3} \div 9^{-1.8}) \cdot 9^{7.6}}$

    A. 9   B. $9^{8.42}$ C. $9^{2.34}$ D. $9^{1.62}$ E. NOTA

28. Find the positive value, in simplest form, of $\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \cdots}}}}$

    A. 3   B. 4   C. 6   D. 12   E. NOTA

29. What is the sum of the solutions of $\log_3(x + 2) + \log_3(x - 4) = 3$?

    A. 2   B. 3   C. 7   D. 12   E. NOTA

30. Find the sum of the first 25 terms of an arithmetic series IF $A_{13} = 97$.

    A. 4850   B. 2425   C. 2597   D. not enough information   E. NOTA

TIE-BREAKERS:

1. Determine the equation of the inverse of $y = (x - 7)^3 - 6$.

2. What characteristic is possessed by any odd function?

3. What is the difference of the focal radii for $\dfrac{(x+3)^2}{24} - \dfrac{(y-5)^2}{10} = 1$?