OpShort

Mu Alpha Theta

Maine '95

Short Cuts

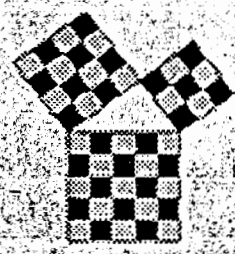
GENERAL INSTRUCTIONS:

1.) Do not begin the test until told to do so.

2.) Use a #2 pencil. Be sure that all Scantron ovals are fully filled in and that all erasures are complete.

3.) Please PRINT your name and school in the name blank and the topic of this test in the subject blank on your Scantron answer sheet.

4.) There will be limited calculator usage on this test. Specific calculators are listed below:
Radio Shack: EC-4008
Casio: FX-82, FX-115, FX-250, FX-300, FX-570, and FX-991 series
Sharp: EL-506 (except EL-506D), EL-509, EL-520, EL-531, and EL-546 series
Texas Instruments: TI-25, TI-30, TI-31, TI-34, TI-35, and TI-36 series



5.) If there is a need for a tie breaker, we will call you back. All tie breakers will be based on content of the test, speed, and accuracy.

1. A regular polygon has an exterior angle of 8°. How many sides does this polygon have?

a. 25 b. 27 c. 36 d. 45 e. not given

2. Solve for x: $\log_{10}(\log_2(\log_4 x)) = 0$.

a. 2 b. 4 c. 8 d. 16 e. not given

3. Which of the following is a factor of $x^3 + 4x^2 - 15x - 18$?

a. x + 9 b. x - 1 c. x - 3 d. x - 6 e. not given

4. Find the positive numerical value, in simplest form, of the expression,

$$\sqrt{12 + \sqrt{12 + \sqrt{12 + \sqrt{12 + \ldots}}}}$$

a. 3 b. 4 c. 6 d. 12 e. not given

5. Find the remainder when $x^{101} - 4x^3 + 5x^2 - 5$ is divided by x - 1.

a. 4 b. 3 c. -3 d. -4 e. not given

6. A circle is inscribed in a right triangle whose legs are 8 and 15. What is the radius of the inscribed circle?

a. 3 b. 4 c. 5 d. 6 e. not given

7. Simplify: $(1-i)^{20}$

a. -1024 b. -1024i c. 1024i d. 1024 e. not given

8. If $x^2+y^2=10$ and $(x-y)^2=2$, find the numerical value of xy.

a. 8 b. -8 c. 4 d. -4 e. not given

9. Two successive discounts of 20% and 20% are followed by two successive increases of 20% and 20%. What single percentage change would produce the same result? State your answer to the nearest tenth of a percent and indicate whether it is a discount or an increase.

a. no change b. 8.2% increase c. 7.8% increase d. 8.2% decrease
e. not given

10. Given Triangle ABC. Side AB is extended to D. Angle CBD = (2x + 70), Angle CAB = (3x -5) and Angle ACB = (x + 60). Find the numerical measure of Angle CBA. All angles are in degrees.

a. 75 b. 80 c. 85 d. 90 e. not given

11. Find the measure of the smaller angle formed by the hour hand and minute hand of a clock at 6:42. All answers are in degrees.

a. 48 b. 51 c. 55 d. 59 e. not given

12. Calculate the sum of the reciprocals of the roots of the equation:

$$x^4 + 4x^3 - 8x^2 + 16x - 32 = 0.$$

a. -2 b. -.05 c. 0.5 d. 2 e. not given

13. In how many zeros does 100! end?

a. 20 b. 24 c. 35 d. 36 e. not given

14. If $x = \sqrt{3+2\sqrt{2}} - \sqrt{3-2\sqrt{2}}$ then x in simplied form is

a. 6 b. 4 c. 2 d. 0 e. not given

15. Find the value of $|40 + 9i|$.

a. $40 + 9i$ b. $40 - 9i$ c. $-40 - 9i$ d. $-40 + 9i$ e. not given

16. If $P(x) = 3x^4 - ax^2 + bx - 2$ is divided by $x + 2$, the remainder is 34. When $P(x)$ is divided by $x - 3$ the remainder is 271. Find $2a + b$.

a. -10 b. -2 c. 2 d. 10 e. not given

17. Five tests scores were lost, but a summary of those scores indicated the mode was 90, the median, 85, and the mean, 83. If the grades were integers from 0 to 100 (inclusive), what is the lowest possible grade from the missing scores?

a. 50 b. 55 c. 66 d. 68 e. not given

18. If the interior angle of a regular convex polygon is thirteen times as larges as the exterior angle, how many diagonals has the polygon?

a. 299 b. 324 c. 350 d. 377 e. not given

19. When $(9x^2 - 6xy + y^2)^5$ is expanded and written in polynomial form with integral coefficients in descending order. The sum of the coefficients is ...

a. 256 b. 512 c. 1024 d. 2048 e. not given

20. What is the sum of the elements in the twelfth row of Pascal's Triangle?

a. 2047 b. 2048 c. 4095 d. 4096 e. not given

21. In a circle, a quadrilateral is inscribed. If the quadrilateral is ABCD with AB = 20, BC = 15, CD = 25, and AD = 20, and AC = 25, Find the numerical length of the other diagonal BD.

a. 18 b. 24 c. 27 d. 32 e. not given

22. How many positive integers less than 2000 are divisible by neither three or seven?

a. 810 b. 954 c. 1049 d. 1144 e. not given

23. If k and m are roots of $3x^2 - 4x - 1 = 0$ the value of $\log_3 k + \log_3 m$ is

a. -2 b. -1 c. 1 d. 2 e. not given

24. Find the area of the quadrilateral whose vertices have coordinates of
A(8, 2), B(-4,6), C(-3, -7), D(5, -3)

a. 90 b. 120 c. 150 d. 180 e. not given

25. Three tangent spheres A, B, and C are also tangent to line m. Sphere C is nested between spheres A and B. If the radius of Circle A is 24 and the radius of circle B is 12. What is the shortest numerical distance between the point of tangency of Circle A and line m to the point of tangency of Circle B and line m?

a. 18 b. 24 c. 27 d. 36 e. not given

26. Suppose the roots of $ax^2+bx+c=0$ are r and s. Which one of the following has roots ar + b and as + b?

a. $x^2-bx-ac=0$ b. $x^2-bx+ac=0$ c. $x^2+3bx+ca+2b^2=0$
d. $x^2+3bx-ca+2b^2=0$ e. not given

27. Find a for which a < b and $\sqrt{1+\sqrt{21+12\sqrt{3}}} = \sqrt{a}+\sqrt{b}$.

a. 1 b. 3 c. 4 d. 12 e. not given

28. Given a circle with two intersecting chords, AB and CD, at point P. AP = 2, PB = 6, CP = 3 and PD = 4. Find the diameter of this circle.

a. $2\sqrt{62}$ b. $3\sqrt{7}$ c. 8 d. $\sqrt{65}$ e. not given

29. How many terms are there in the expansion $(a + b + c + d)^{10}$?

a. 11 b. 15 c. 286 d. 10000 e. not given

30. $S = \frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \left(\frac{1}{5} + \frac{2}{5} + \frac{3}{5} + \frac{4}{5}\right) + \ldots + \left(\frac{1}{100} + \ldots + \frac{99}{100}\right), S = ?$

a. 1000 b. 1575 c. 2200 d. 2475 e. not given