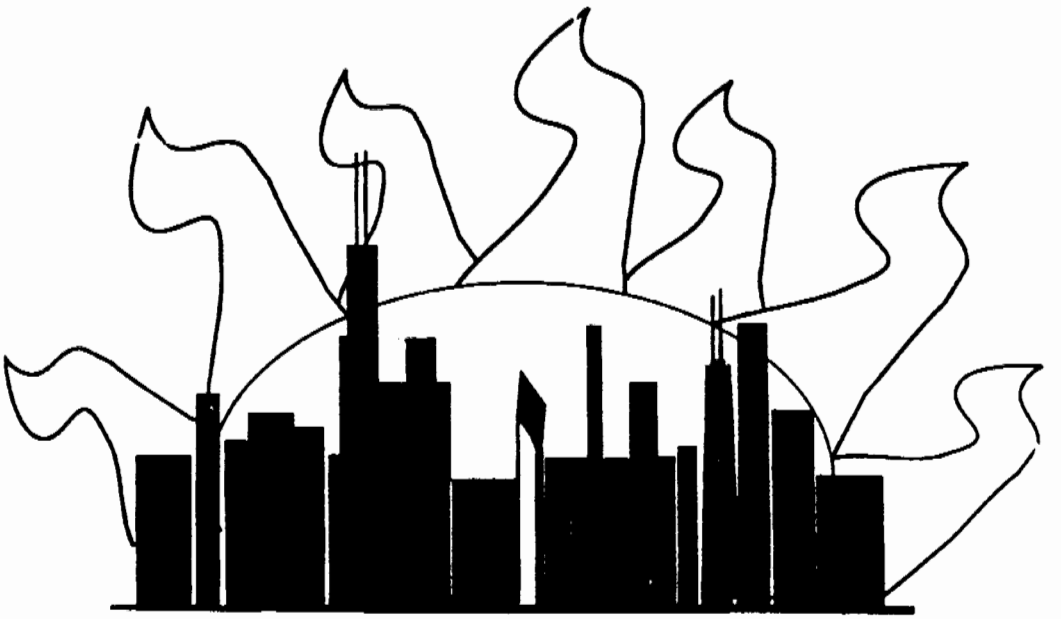
**Alpha Division** **Topic Test 2**

# Complex Numbers



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## Level - Alpha

**General Instructions:**

Unless otherwise stated all answers should be written as decimals.
If you are asked to give your answer as a fraction, please give your answer in *a/b* form where *a* and *b* are relatively prime.

For all problems on this test, $i = \sqrt{-1}$

**Questions**

1. Solve the following equation for $x$. Express your answer in the form $a+b$, where $a$ and $b$ are real numbers.

$$\frac{4-3i}{2+i}x + 3i - 6 = \frac{3+i}{2-i}x$$

2. If $i$ is the imaginary unit, find the decimal equivalent of $\left(\frac{4}{5}\right)^{i^{434}}$.

3. Determine $|4+3i|$

4. If $x = y^2$ and $y = x^2$, find the least non-negative value of $x^2 + x + 1$, where $x$ and $y$ are complex numbers.

5. Given that $i^2 = -1$, for how many integers, $n$, is $(n+1)^4$ an integer?

6. If $a + bi$, with $a$ and $b$ real numbers, is the complex conjugate of the multiplicative inverse of the product $(2-3i)(-2-4i)$, find the value of $b$. Round your answer to the nearest thousandth.

7. Given that one root of $f(x) = x^3 - 4x^2 - 2x + 20$ is $-2$, find the larger of the other roots.

8. Find the value of $(-i)^{4n-1}$ when $n$ is a negative integer.

9. Suppose

$$[2-(-2+i\sqrt{3})-(-2-i\sqrt{3})][2+(-2+i\sqrt{3})^2+(-2-i\sqrt{3})^2][2-(-2+i\sqrt{3})^4-(-2-i\sqrt{3})^4] = 2^k 3^m,$$

find the value of $k$, if $k$ and $m$ are integers.

10. If two roots of the polynomial equation $x^6 - 6x^5 + 14x^4 - 22x^3 + 25x^2 + 8x = 60$ are $2i$ and $2-i$, then find the sum of the real roots.

11. Simplify $\dfrac{(2+i)^2}{2-i} - \dfrac{(2-i)^2}{2+i}$

12. Simplify $(\sqrt{9+40i} + \sqrt{9-40i})^2$

13. Simplify $(-1+i)^7$. Express your answer in the form $a + bi$.

14. Evaluate $(1 - i\sqrt{3})^6$

15. If $w = 5 - i\sqrt{7}$ and $\overline{w}$ is the conjugate, then evaluate $w^2 + 2w\overline{w} + \overline{w}^2$ .

16. Find the product of the ordered pair of real numbers $(x, y)$ for which
$$x + xi + y - yi = -1 + 7i.$$

17. For how many integers $c \leq 50$ will the solutions of the equation $x^2 - 4x + c = 0$ be complex conjugates of the form $a + bi$, where $a$ and $b$ are positive integers?

18. Let $z$ be a complex number and $\overline{z}$ be its conjugate. Find the least value of $z$ for which $z\overline{z} = 5$ and $z^2 + \overline{z}^2 = 6$ . Write your answer in the form $a + bi$.

19. Find the sum of the absolute values of the imaginary roots of
$$8x^4 - 6x^3 + 15x^2 - 18x - 27 = 0$$

20. If $e^x = i^i$, find $x$. Note: $i^i$ is a real number, even though $i$ is imaginary.