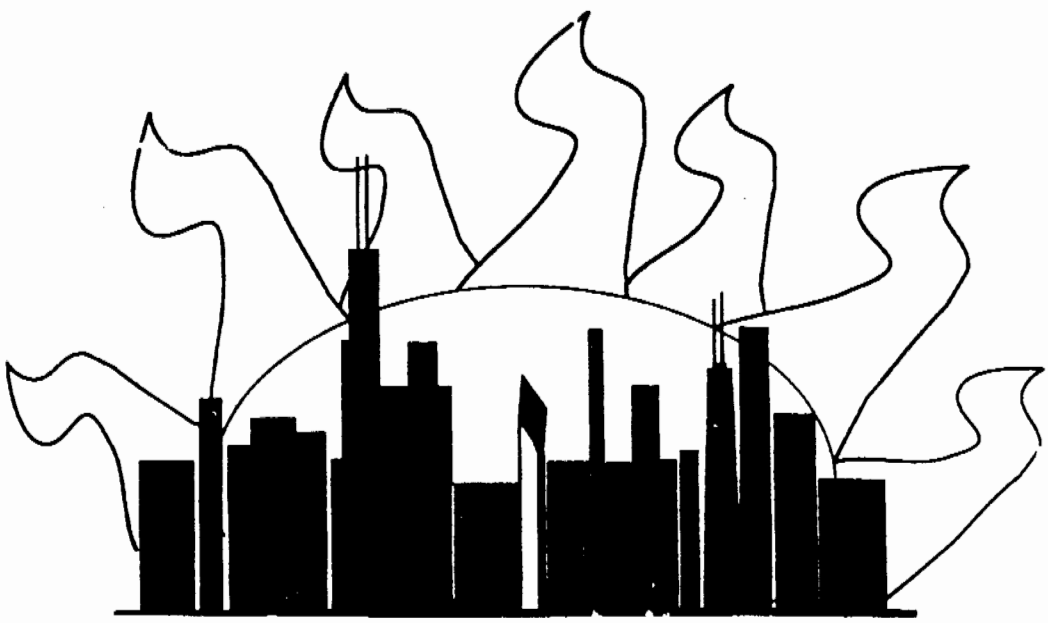
Alpha Division **Topic Test 2**

Combinatorics



Mu Alpha Theta National Convention
Chicago 1998

Level - Alpha

General Instructions:

Unless otherwise stated all answers should be written as decimals.
If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.

Questions

1. In the local tennis tournament, the champion is to be determined from among 100 competitors by elimination. As soon as one of the competitors loses a match, the competitor joins the ranks of the spectators. How many matches are needed in order to determine the winner?

2. How many four-element subsets of {1, 2, 3, 4, 5, 6, 7} have a 1 as an element but do not have 7 as an element?

3. A group of six students is choosing each other's names for a grab bag gift. All six names are put in a hat and then each student draws out one name. What is the probability that exactly four of the students get their own names? Give your answer as a reduced fraction.

4. A committee is formed by selecting 3 people from a group of 5 men and 4 women. Find the probability that the committee contains exactly 1 man and 2 women. Give your answer as a reduced fraction.

5. After a meeting where 10 Mu Alpha Theta members sat around a circular table, each person shook hands with everyone else except for the people who sat on either side of them. How many handshakes took place?

6. In how many ways can a careless person place four letters in four envelopes so that no one gets the correct letter?

7. Find the positive integer n satisfying $\binom{n}{1} + \binom{n}{2} + \binom{n}{3} = 231$ where $\binom{n}{1}$, $\binom{n}{2}$, and $\binom{n}{3}$ are binomial coefficients.

8. The only child present in a room noticed that each man shook hands with each of the other men while each woman hugged each of the other women present. If there were 15 handshakes and 21 hugs exchanged, how many people were in the room?

9. Sara has a penny, a nickel, a dime, and a quarter. What is the total number of non-zero amounts of money which can be formed from some or all of these coins?

10. In the school band five students each own their own trumpet. In how many different ways can exactly three of the five students take home the wrong trumpet, while the other two take home their own trumpet?

11. 852 digits are used to number the pages of a book consecutively starting with 1. How many pages are in the book?

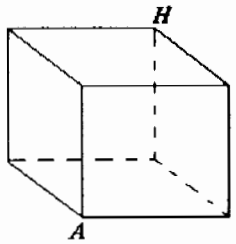
12. Three judges for a talent show have to vote publicly on three performers A, B, and C, listing their order of preference. In how many ways can the judges vote so that any two of them agree in their order of preference, while the third differs?

13. In how many different ways can 35 cents be made up of coins of the types currently being minted in the United States?

14. The squash season is nearing its end and the current individual standings are shown in the table below. Each of the 8 players still must play 28 games, 4 with each of the other players. How many players still have a theoretical chance to at least tie for the championship?

Player	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Games Won	92	91	90	71	67	66	44	69
Games Lost	48	49	50	69	73	74	96	101

15. The vertices of a unit cube (one whose edges are each one unit) are labeled so that A and H are diametrically opposite each other. How many paths of length 3 are there from A to H that follow the edges of the cube?



16. There are 18 people in the final round for a grand prize. The 18 must stand in a circle and be counted for elimination. Starting with number 1, every seventh contestant will be eliminated until one remains to win the prize. In what original position should you stand to win the contest?

17. How many different positive integers are factors of the number which is the product of 33554432, 1728 and 531441?

18. Five letters and envelopes are addressed to five different people. In how many ways can the letters be placed into the envelopes so that every envelope gets a letter but at most one letter is placed in the correct envelope?

19. Find the exact value of $\frac{99! + 100! + 101!}{101!}$ Write your answer in decimal form.

20. How many different ways are there to arrange all of the letters *ILLINOIS*?