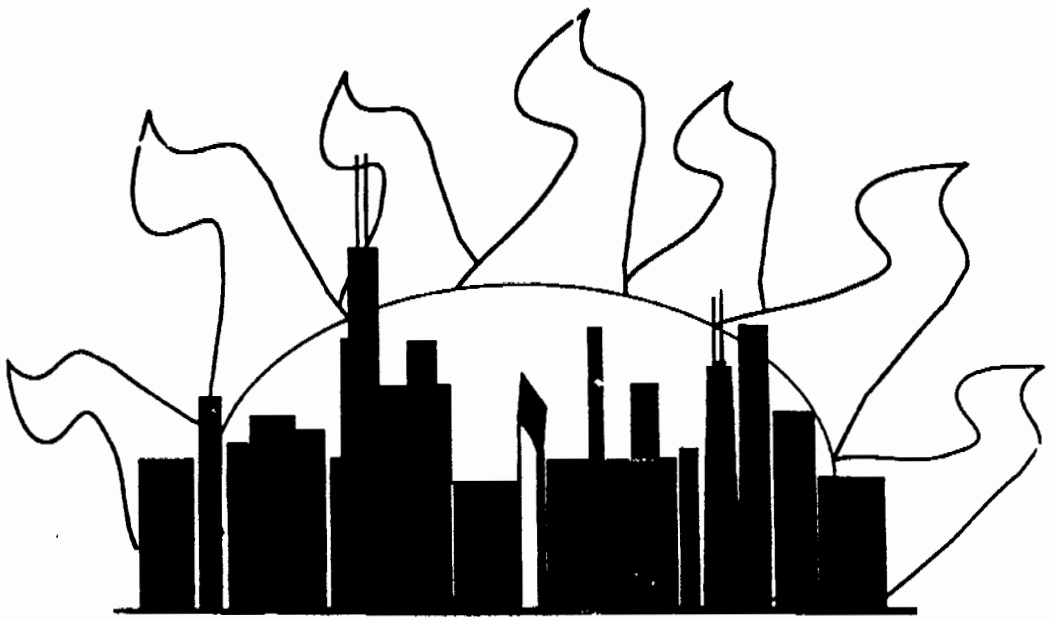
Alpha Division **Topic Test 1**

Finite Differences



Mu Alpha Theta National Convention
Chicago 1998

Alpha Level

General Instructions:

Unless otherwise stated all answers should be written as decimals.
If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.

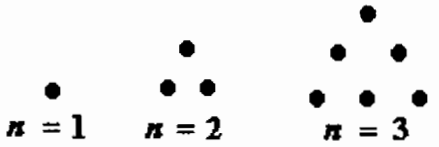
Questions

1. A large cube was put into a vat of paint. The cube was then cut into 1000 cubes of equal size. How many of the smaller cubes would not be painted at all?

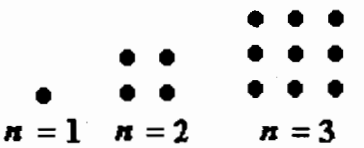
2. If the recursive relationship is $F(n) = F(n-1) + 2n$ and $F(1) = 2$, $F(2) = 6$, $F(3) = 12$, determine the functional relationship in terms of n. Write your answer as $F(n) =$ answer.

3. What is the smallest number > 1 that is both a triangular number and a square number?

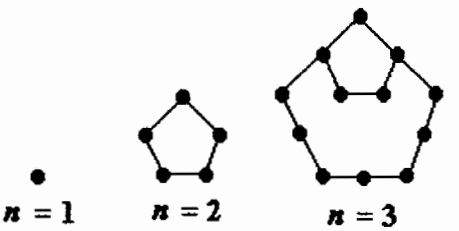
Triangular numbers



Square numbers

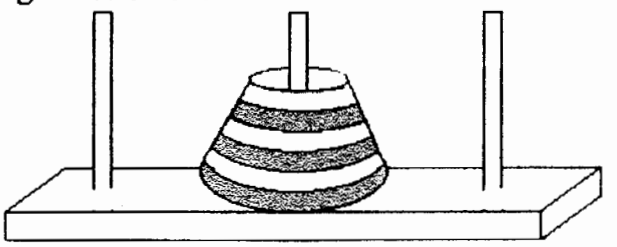


4. How many dots are in the pentagonal number when $n = 100$?



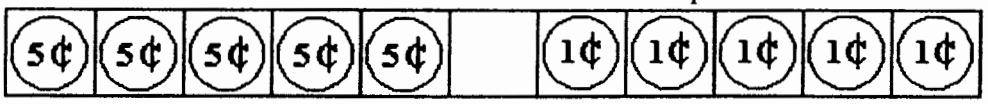
5. What is the sum of the series $1\cdot2 + 2\cdot3 + 3\cdot4 + 4\cdot5 + 5\cdot6 + \ldots + 49\cdot50$?

6. The object of the ancient puzzle, Tower of Hanoi, is to transfer the tower of discs to either of the two vacant pegs in the fewest possible moves. You may move only one disc at a time and you may not place a larger disc on one that is smaller.



What is the minimum number of moves necessary to move n discs?

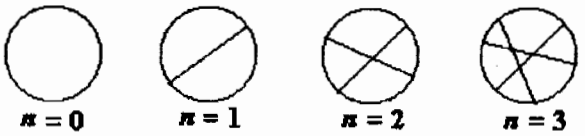
7. In order to get the five pennies to the squares where the nickels are and vice versa you can a) slide a coin to an empty space next to it or b) jump one coin (only one) of the other denomination. Determine the minimum number of moves (slides + jumps) it will take to get the pennies where the nickels are and the nickels where the pennies are.



8. Find the *n*th term in the sequence 4, 11, 18, 25, ...

9. During the Civil War the men in Company A stacked their cannonballs in a triangular pyramid. The men in Company B stacked their cannonballs in a square pyramid. Each company had a stack of cannonballs that contained 10 layers. How many more cannonballs did Company B have than Company A?

10. A chord is a line segment joining two points on a circle. n is the number of chords and R is the maximum number of regions formed by a given number of chords in a circle

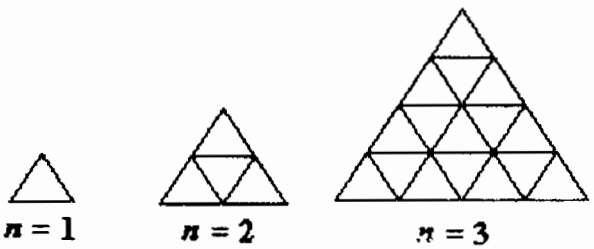


What is the maximum number of regions formed by 60 chords?

11. Determine T in terms of n.

n	1	2	3	4	5	6 ... n
T	2	8	18	32	50	72

12. An equilateral triangle can be divided into smaller equilateral triangles. In this problem count only point-up triangles but count point-up triangles of all sizes.

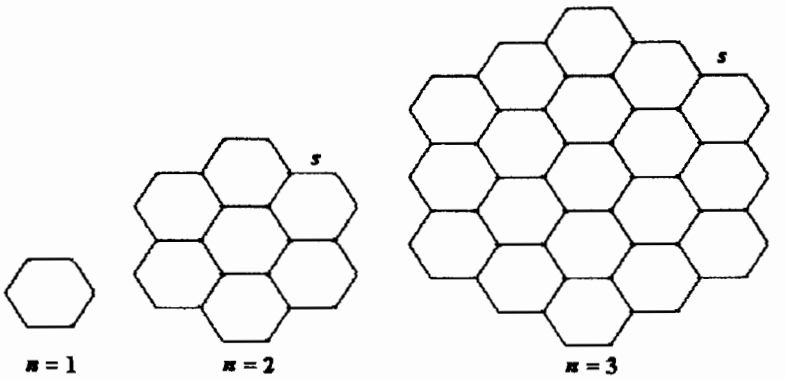


How many point-up triangles are in an equilateral triangle with n = 50 triangles on a side?

13. Determine T in terms of n.

n	1	2	3	4	5	6 ... n
T	1	3	5	7	9	11

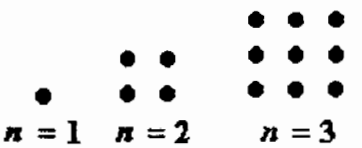
14. Bees are clever engineers. The cross section of a honeycomb is a mosaic of hexagons with each corner surrounded by three hexagons. To find the volume of a honeycomb we need to multiply the base by the height. If $n = 3$, $s = 2$ and $h = 4$, what is the volume of the honeycomb?



15. Determine T in terms of n.

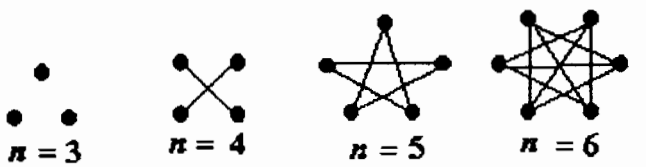
n	1	2	3	4	5	6 ... n
T	8	11	16	23	32	43

16. The Orchard brothers always plant their apple trees in square arrays.



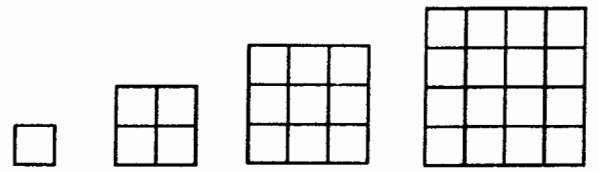
This year they planted 31 more apple trees in their square orchard than last year. If the orchard is still square, how many apple trees are in the orchard this year?

17. A diagonal of a polygon is a line segment that connects any two non-adjacent vertices. In these examples n represents the number of sides of the polygon.



How many diagonals will there be in a polygon that has 100 sides?

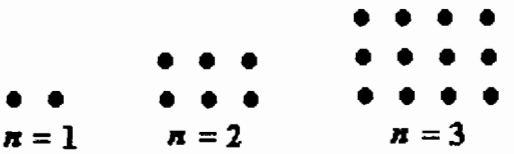
18. A large square can be divided into many smaller squares. Count *all* squares. n represents the number of units on a side of the large square.



How many squares would there be if $n = 50$?

19. Given any two non-negative consecutive integers, n and $n + 1$, express the absolute value difference of their squares in terms of n.

20. Oblong numbers are numbers that can be represented in a singular array having one dimension one unit longer than the other dimension.



Express the general rule for the n^{th} oblong number.