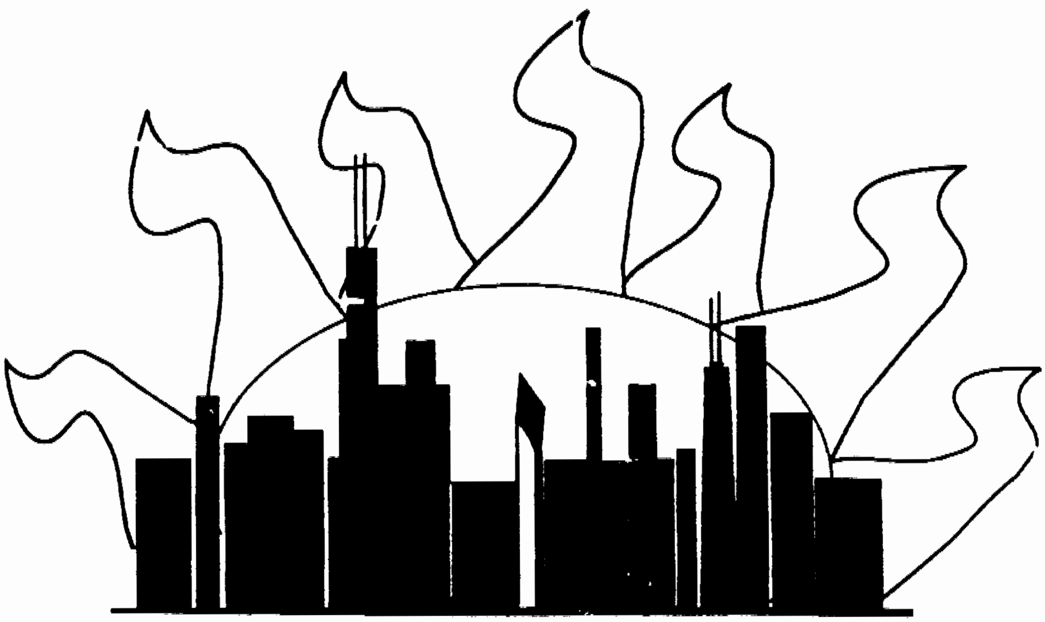
Alpha Division **Topic Test 2**

Sequences & Series



Mu Alpha Theta National Convention
Chicago 1998

Alpha Level

General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.

Questions

1. Starting with 1, at most how many consecutive integers can be added together before the sum exceeds 200.

2. If $3 \le y \le 22$, for how many ordered pairs of positive integers (x, y) does y exceed x by at least 2?

3. If five geometric means are inserted between 8 and 5832, what is the fifth term in this geometric sequence?

4. Two positive numbers may be inserted between the numbers 3 and 9 such that the first three are in geometric progression, while the last three are in arithmetic progression. What is the sum of these two positive numbers? Give your answer in decimal form.

5. The radii, in feet, of a sequence of cardboard circular disks form a geometric progression: $8, 4, 2, 1, \ldots$. What is the radius, in feet, of the first circle in the progression that will fit completely on a square piece of paper with an area of 36 square inches? Give your answer as a simplified fraction in lowest terms.

6. Calculate the sum of all integers between 1 and 200 inclusive which are multiples of 3 and/or multiples of 5.

7. Find the smallest integer n such that the sum $1+3+3^2+3^3+\cdots+3^n$ exceeds 1,000,000.

8. What is the sum of the first 80 positive odd integers subtracted from the sum of the first 80 positive even integers?

9. In the series $20^2-19^2+18^2-17^2+\cdots+2^2-1^2$, the signs alternate between squares of consecutive integers. Compute the sum of this series.

10. A series of 7 books was published at 9 year intervals. When the 7th book was published, the sum of the publication years was 13601. In what year was the fourth book published?

11. What is the sum of the coefficients of $(x+1)^{15}$?

12. Suppose a_1, a_2, a_3, a_4 are positive numbers which are consecutive terms in a geometric progression. If $a_1 + a_2 = 10$ and $a_3 + a_4 = 22\frac{1}{2}$, find $a_1 + a_4$. Give your answer in decimal form.

13. In an arithmetic progression, the 25^{th} term is 2552 and the 52^{nd} term is 5279. Find the 79^{th} term.

14. Compute $1^2 - 2^2 + 3^2 - 4^2 + \cdots + 97^2 - 98^2 + 99^2$.

15. Find this sum rounded to three decimal places: $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \cdots + \frac{1}{97 \cdot 99} + \frac{1}{99 \cdot 101}$.

16. Find the product of the first 10 terms of a geometric series whose first term is 1 and whose tenth term is 2. Give your answer in integral form.

17. Evaluate: $1^2 - 2^2 + 3^2 - 4^2 + \cdots + 399^2$.

18. What is the sum of this infinite series: $\frac{1}{7} + \frac{2}{7^2} + \frac{1}{7^3} + \frac{2}{7^4} + \cdots$? Give answer as a simplified fraction in lowest terms.

19. A sequence of five consecutive positive integers is considered to be <u>neat</u> if the first integer is a multiple of 2, the second is a multiple of 3, the third is a multiple of 4, the fourth is a multiple of 5 and the fifth is a multiple of 6. How many <u>neat</u> sequences are there less than 1000?

20. Let A_n be the sum of the first n terms of the geometric series $704 + \frac{704}{2} + \frac{704}{4} + \cdots$ and B_n be the sum of the first n terms of the geometric series $1984 - \frac{1984}{2} + \frac{1984}{4} - \cdots$. Compute the value of n (with $n \geq 1$) for which $A_n = B_n$.