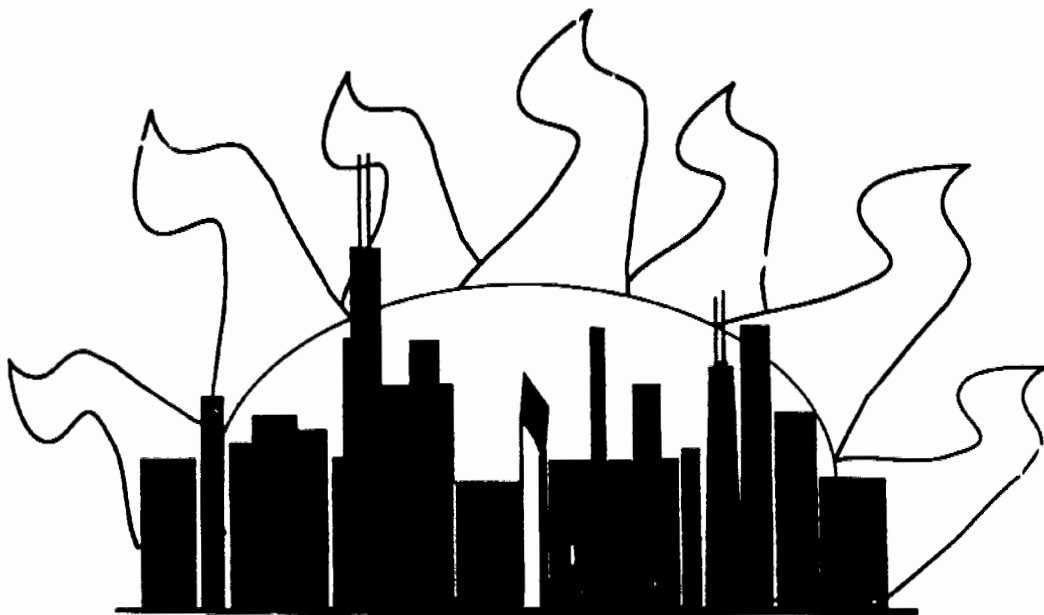


Alpha Division

Topic Test 3

Trigonometry



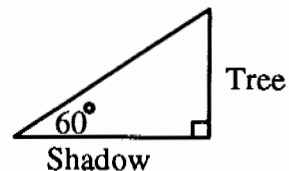
**Mu Alpha Theta National Convention
Chicago 1998**

General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.

Questions

1. A tree casts a shadow that is 22 feet long. The angle of elevation of the sun is 60° . Find the height of the tree in feet. Give your answer to the nearest tenth of a foot.



2. Assume $\frac{\pi}{2} < x < \pi$ and $\cos x = c$. Find $\arccos(-c)$, in radians. Note: An alternate notation for $\arccos(-c)$ is $\text{Arc cos}(-c)$.
3. Find the value of x , $0 \leq x \leq 2\pi$, that satisfies $2(\sin^2 x) + \sin x = 1$ and $|2x - 3| > 4$.
4. Find the smallest positive real value of x such that a rectangle with dimensions $\cos x$ by $\sin x$ has an area of .2834. Give your answer rounded to four decimal places.
5. Compute $(\sin 15^\circ + \sin 75^\circ)^6$. Give your answer as a simplified fraction in lowest terms.
6. If $\sin(2x) = \tan x$ and $\frac{\pi}{2} < x < \pi$, what is the exact value of $\cos x$?
7. If $\tan x + \cot x = \frac{144}{25}$, find the numerical value of $\frac{1}{\tan x} + \frac{1}{\cot x}$. Write your answer as a decimal to the nearest hundredth.
8. Find, in degrees, the least positive angle x for which $\left(2^{(\sin^2 x)}\right)\left(2^{(\cos^2 x)}\right)\left(2^{(\tan^2 x)}\right) = 2^2$.
9. Find the exact value, in radians, of the sum of all values of θ , $0 \leq \theta < 2\pi$, such that $\sin(2\theta) = \cos(2\theta) - 1$. Give answer as a simplified fraction in lowest terms.

10. Find the smallest positive value of x , in degrees, for which $\sin^4 x + \cos^4 x = \frac{5}{8}$.
11. Simplify: $\sin^2 \theta + \cos(2\theta) - \cos^2 \theta - \sec^2 \theta + \tan^2 \theta$.
12. Find the largest value of θ for which $0 < \theta < 2\pi$ and $\sin \theta \cos \theta = -\frac{\sqrt{3}}{4}$. Give an exact answer as a simplified fraction in lowest terms.
13. Find the smallest positive value for x if $(\sin x \cos x - \sin^2 x)(\cos^2 x + \cos x \sin x) = \frac{1}{8}$.
Give an exact fractional answer in radians.
14. You are given one coin with diameter 5 cm. and a large supply of coins with diameter 2cm. What is the maximum number of the smaller coins that may be arranged around the larger coin so that all the smaller coins are tangent to the larger coin without any overlap?
15. For all real values of x for which it is defined, $f(x) = \cot\left(\frac{1}{4}x\right) - \cot x$ can be written in terms of the sine function as $f(x) = \frac{\sin(kx)}{\left(\sin\left(\frac{1}{4}x\right)\right)(\sin x)}$. Find the value of k . Express your answer as a simplified fraction in lowest terms.
16. Find the sum of the amplitude and the period of $y = -46(\sin(3.2x + 22.4))$. Give your answer rounded to four significant digits.
17. Find the value of $\sin^2 1^\circ + \sin^2 2^\circ + \sin^2 3^\circ + \dots + \sin^2 90^\circ$. Give your answer as a simplified fraction in lowest terms.
18. Point A has polar coordinates $(3.7, 37.4^\circ)$. Point B has rectangular coordinates $(1.077, 5.292)$. Find the distance from point A to point B. Give your answer rounded to four significant digits.
19. Find the sum of all roots of the equation $2^x = \sin(x + 2)$ which are greater than -2.5 . Give your answer rounded to three significant digits.
20. Find $x + y$ for which, in $\triangle ABC$, $\sin A : \sin B : \sin C = 4 : 5 : 6$ while the ratio $\cos A : \cos B : \cos C = x : y : 2$.