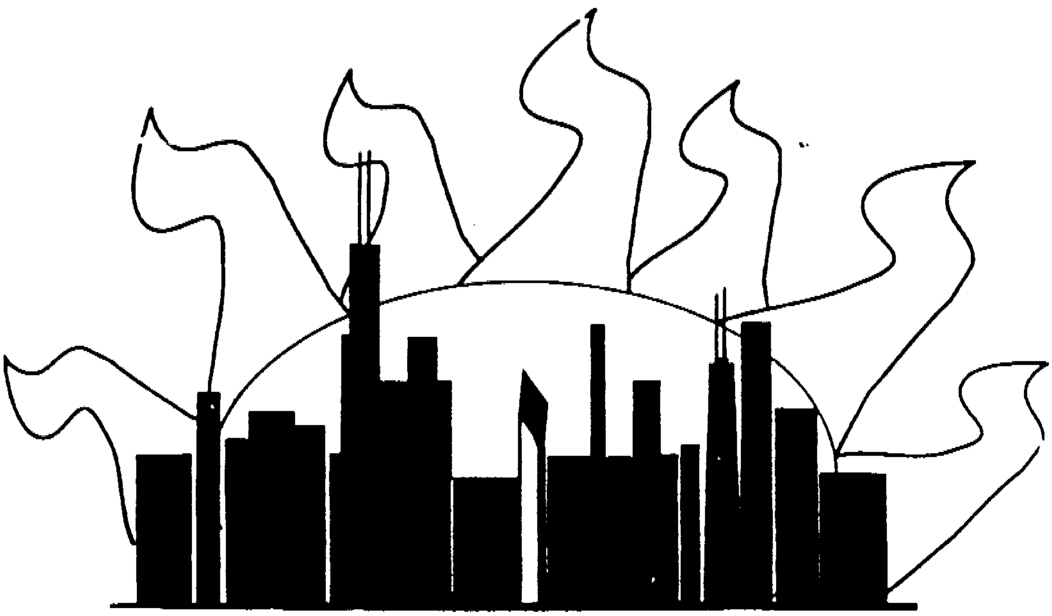
Individual Test



Mu Alpha Theta National Convention
Chicago 1998

Note to Alpha and Theta Students: We are using the same test for all of the three levels, Mu, Alpha, and Theta so there may be some questions which you will have difficulty answering. Please attempt as many as you can. The awards for each level will be determined by the scores of students who are taking the test at that level.

General Instructions:

Unless otherwise stated all answers should be written as decimals.
If you are asked to give your answer as a fraction, please give your answer in *a*/*b* form where *a* and *b* are relatively prime.

Questions

1. What is the value of R?

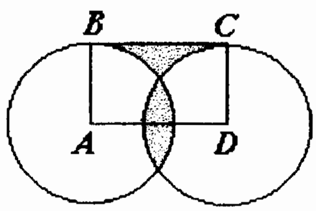
   Q + M = C
   C + K = R
   R + Q = S
   M + K + S + 20
   Q = 4

2. Find the number of ways that five large books, four medium size books and two small books can be placed on a shelf so that all of the books of the same size are together?

3. Which regular polygon has the same number of diagonals as it has sides? Give your answer as the number of sides.

4. In the course of manufacturing bars of soap that have rounded edges, the cutting machine produces scraps. The scraps from 11 bars of soap can be made into one extra bar. How many complete bars can be made from the scraps after cutting 250 bars of soap?

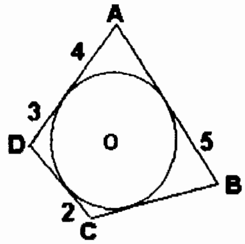
5. From "When Does a Dog Become Older Than Its Owner?", an article by Anne Quinn and Karen Larson in *The Mathematics Teacher*: The premise of this problem is that since dogs supposedly age seven times as quickly as humans, at some point the dog will become "older" than its owner. Alisa wanted to determine exactly on which day this milestone would occur for her dog and her dog, Taffy, so that they could celebrate the occasion. These are the basic facts: Alisa was born on August 19, 1975 and Taffy was born on October 5, 1977. For every year that Alisa aged, Taffy aged seven equivalent years. On what date are, or were, Alissa and Taffy "the same age"? Write your answer as the numeric values of the month, day, last two digits of the year, i.e: January 1 1998 would be written as 1198.



6. In the diagram to the right, congruent radii $\overline{AB}$ and $\overline{CD}$ intersect tangent $\overline{BC}$. If the shaded regions have equal areas, and if $AB = 1$, find the area of quadrilateral $ABCD$. Give your answer as a reduced fraction.

7. At what latitude north of the equator, in degrees, is the distance around the earth half of what the distance around the earth is at the equator?

8. Quadrilateral ABCD is circumscribed about a circle O. Find the perimeter of ABCD.



9. One barrel of pickles weighs 120 pounds, while one box of strawberries weighs 36 pounds. What is the total weight, in pounds, of one crate of cherries and one sack of potatoes, when the sack of potatoes and the box of strawberries together weigh as much as the barrel of pickles and the crate of cherries? Two crates of cherries weigh as much as one box of strawberries.

10. Consider all lines through the point $(-3, 2)$ such that the sum of the x and y intercepts equals twice the slope. What is the maximum slope of all such flies?

11. A person is assumed to be of age n until his or her $n + 1^{st}$ birthday. If two persons, each less than ten years old, are selected at random (i.e. their ages are between 0 and 9, inclusively, and each of these ages is equally likely), what is the probability that the sum of their ages is at most 7? Give your answer as a reduced fraction?

12. On the planet Oberon, there are as many days in a week as there are weeks in a month. The number of months in a year is twice the number of days in a month. If there are 1250 days in an Oberon year, how many months are there in an Oberon year?

13. A projectile is fired from the ground with a muzzle speed of 300 meters per second. The projectile is fired at an angle of 40° with the ground. Assuming the acceleration due to gravity of 9.8 meters per sec^2, how far downrange does the projectile travel before striking the ground? Give your answer to the nearest meter.

14. 15! which is $15 \cdot 14 \cdot 13 \cdot \ldots \cdot 1$ ends with m zeros when the number is written in base ten and has n zeros when the number is written in base twelve? What is the sum of $m + n$?

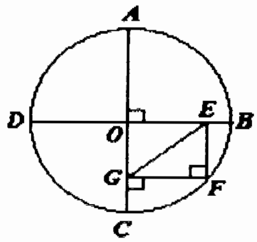
15. In an arithmetic progression, the 25th term is 2552, and the 52nd term is 5279. What is the 79th term?

16. A cylindrical bold has its outer edge of the bolt on a cylinder ½ inch in diameter. There are fifty thread spaces per inch. Find the measure of the angle at which the thread rises. Find the measure to the nearest ten-thousandths of a degree.

17. A square is inscribed in an octagon using segments that join every other vertex of the octagon. If the length of the square is 4, find the area, in square units, of the octagon. Give an exact answer.

18. The sum of three positive integers is 50. Find the greatest possible product of those three numbers.

19. AC and BD are diameters pf circle O. What is the radius of the circle in inches? $EG = 8$ in,, CG = 3 in.



20. What is the width of the largest rectangle with length 16 in. that you can cut from a circular piece of cardboard having a radius 10 in?

21. $A = \begin{bmatrix} 0 & 1+i & 1+2i \\ 1-i & 0 & 2-3i \\ 1-2i & 2+3i & 0 \end{bmatrix}$ where $i = \sqrt{-1}$. Determine $|A|$.

22. $TRAP$ is a trapezoid with $\overline{TR}$ the lower base. m∡$T = 90°$. m∡R = 60°, $AR = AP = 8$. Find the area of the trapezoid. Give an exact answer.

23. What is the sum of all real values of x that satisfy $(x+5)^{x-2} = 1$?

24. Given triangle ABC with m∡$BAC = 30°$ and $BC = 6$. Find the diameter of the circle which circumscribes the triangle.

25. Find the limit as n approaches infinity. $\frac{1}{n^3} + \frac{4}{n^3} + \frac{9}{n^3} + \ldots + \frac{n^2}{n^3}$ Give your answer as a reduced fraction.