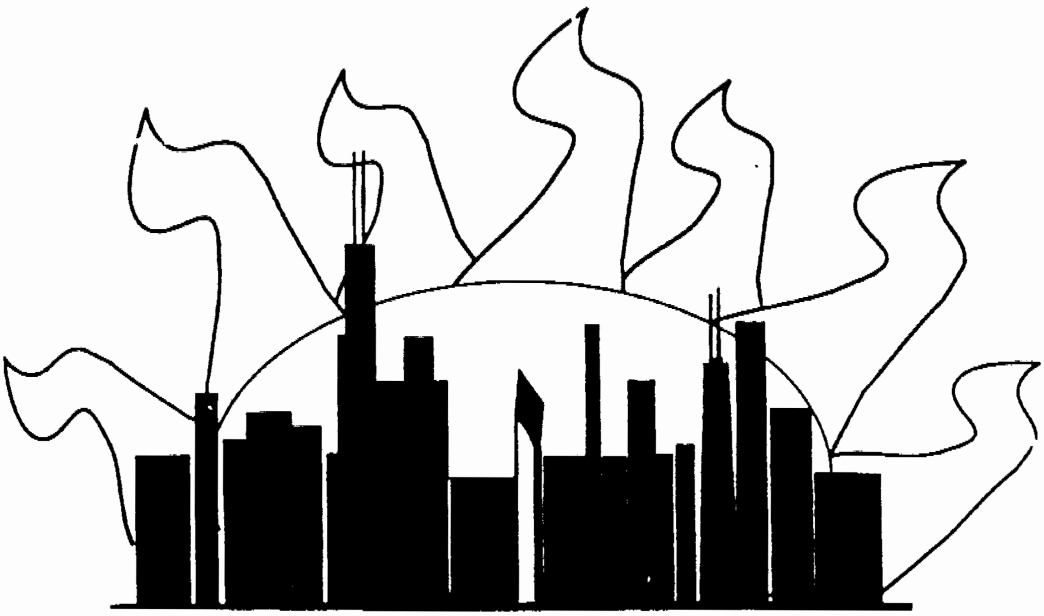
Mu Division **Topic Test 3**

Advanced Calculus



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General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.
3. Vectors are indicated by boldface notation.
4. $\frac{\partial y}{\partial x}$ denotes the partial derivative of y with respect to x.

Questions

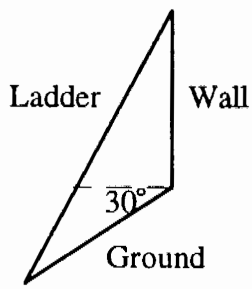
1. Find the equation of the tangent line to the graph of $y = -2x^3 + 4x^2 - 7x + 6$ at the point $(2, -8)$. Give as your answer the y-coordinate of the point where the tangent line crosses the y-axis.

2. Determine the sum of the coordinates of the point of inflection of the graph of $y = x^2 (\ln x)$. Give your answer rounded to four significant digits.

3. Find the volume of the solid generated when the portion of the ellipse $4x^2 + 9y^2 = 36$ that is in the first quadrant is rotated around the x-axis.

4. A block of ice measuring 15 cm. × 15 cm. × 20 cm. is melting. If every side is melting at a rate of 1 cm/hr, what is the rate of loss of volume, in cm^3 / hr, at the end of one hour?

5. A ladder 5 meters long rests against a vertical wall so that the foot of the ladder is 120 cm, measured along the ground, from the base of the wall. The ground is not flat but slopes downward at a 30° angle. The ladder starts to slide down the hill with the foot of the ladder moving at a constant rate of 4 cm/sec. How many seconds until the top of the ladder touches the ground?



6. Find $f'(5)$ if $f(x) = x^{(\ln x)}$. Give your answer rounded to four significant digits.

7. For $f(x) = 2x^3 + 6x^2 - 18x + 4$ find the absolute value of the difference between the minimum and maximum values of f on $[-4, 4]$.

8. Calculate $\int_{e^{e^e}}^{e^{e^{e^2}}} \frac{dx}{x(\ln x)(\ln(\ln x))(\ln(\ln(\ln x)))}$. Give answer rounded to four significant digits.

9. If y satisfies the differential equation $y'' = y\,y'$ and $y(0) = -1$, $y'(0) = 1$, find $y(\pi)$.

10. Find $\left.\frac{\partial g}{\partial y}\right]_{\substack{x=2 \\ y=3}}$ if $g(x, y) = x^{y^x}$ with $x, y > 0$. Give answer to the nearest integer.

11. A parabola has its x-intercepts at $x = 0$ and $x = t$ and the y-coordinate of the vertex is 4. If $\mathrm{A}(t)$ represents the area below the parabola and above the x-axis, find a formula for $\frac{d\mathrm{A}}{dt}$. Give your answer as simplified fraction in lowest terms.

12. The base of a solid is a circle of radius 3. The cross sections of the solid in planes perpendicular to the xy-plane, which contains the base of the solid, are regular hexagons with one side as a chord of the circle. Find the volume of the solid.

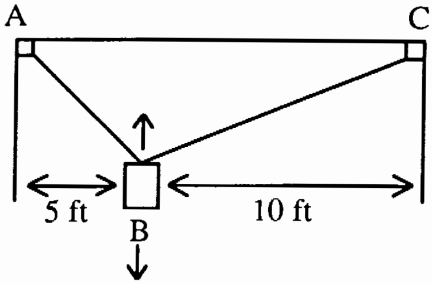
13. Given that $f'(5) = 4$ and $g(x) = 2[f(3x-1)] + 7$. Find one point (x, y) belonging to the graph of $g'(x)$. Give, as your answer, the y-coordinate of that point.

14. What is the area enclosed by the polar curve $r = 3 + 2[\cos 3\theta]$ in the first quadrant? Give your answer rounded to four significant digits.

15. $f(x)$ is a quadratic function satisfying $f'(1) = 3$, $f'(4) = -2$, and $f(2) = 6$. Evaluate $f(0) + f'(0) + f''(0)$.

16. Find $\left.\frac{dy}{dx}\right]_{x=1}$ if $y = \int_{2x}^{x^2+3} (4t^2 - 2t + 1)\,dt$.

17. An elevator consists of a box suspended on a rope as follows: the rope starts on a spindle at A, passes through a pulley on the top of the elevator at B, and then is tied off at point C (see diagram to the right). The box is on a track that keeps it 5 feet from the wall of A and 10 feet from the wall of C. If the car is 12 feet below the roof $(\overline{AC})$ and descending at 1 ft/sec, how fast (in ft/sec) is the spindle at A unwinding rope? Give answer rounded to four significant digits.



18. Find the length of the first two complete rotations of the polar spiral $r = \theta$. Give your answer rounded to four significant digits.

19. A bowl is shaped like the graph of $z = x^2 + y^2$. Water is flowing into the bowl at a rate of 1 cubic unit per second. How fast is the water level rising when it is one unit deep?

20. A tent has corners at (10, 0), (0, 10), $(-10, 0)$ and $(0, -10)$. The height of the tent, at any point (x, y), is $10 - \dfrac{(x^2 + y^2)}{10}$. Find the volume of the tent. Assume that the tent's roof does not sag and forms a planar surface and that the top point of the tent is directly over (0, 0) .