MAΘ - 1998 **Gemini - Mu Level**

1. Let $p(x) = 5x^5 - x^3 + \frac{3}{2}x^2 + \frac{3}{4}x - \frac{1}{4}$ and $q(x) = 2x^2 - x + \frac{1}{2}$. Find the coefficient of x^2 in the simplified expression $p(x)\,q(x)$.

2. A class contains 5 boys and 5 girls. They select seats at random around a circular table that seats 10. Find the probability that at least two girls will sit next to each other.

3. $\dfrac{1}{\dfrac{1}{\dfrac{1}{s-1}-1}-1} - 1 = 1$ and $\dfrac{t+7}{7} = \dfrac{13}{8}$.

   Find the value of the product st.

4. Let $f(x) = 5 - x^2$ and $g(x) = \sqrt{5-x}$. Find the distance between the points $(3, f(3))$ and $(-4, g(-4))$.

5. V is the volume of the solid generated when the triangle formed by $3x + 4y = 60$ and the x- and y-axes is revolved around the y-axis.
   N is the number of primes less than 100.

   Find the value of $\dfrac{V}{N}$.

6. If $r + s = 3$ and $\dfrac{1}{r} + \dfrac{1}{s} = \dfrac{4}{7}$, find the value of $r^2 + rs + s^2$.

7. Consider all positive integer solutions to $3x + 4y = 50$. What is the difference between the largest and smallest values of x that can occur?

8. Find the smallest number x over the interval $0 \le x < 2\pi$ for which the function $y = 8\sin\left(5x - \dfrac{\pi}{2}\right) + 2$ reaches a maximum value.

9. Give the value of A where $2^A = (1 + i)^{200} + (1 - i)^{200}$

10. A house valued at \$90,000 in 1985 was sold for \$250,000 in 1998. Assuming that the value of the house was modeled during that period of time by the exponential function $y = ar^x$, give the value of r to the nearest hundredth.