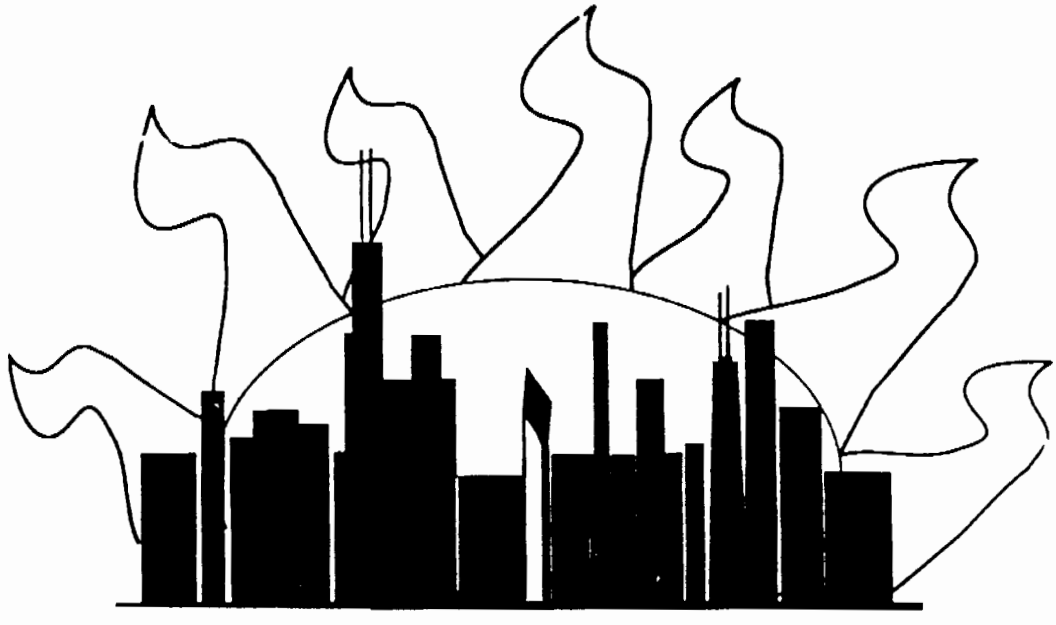
Mu Division **Topic Test 2**

Integral Calculus



Mu Alpha Theta National Convention
Chicago 1998

Mu Level

General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.

Questions

1. Find the smallest positive value of a for which $\int_0^a (3x^2 - 14x + 12)\,dx = 0$.

2. The substitution $u = \cos^{(-1)}(x)$ is made in $\int_{-1}^{1} f(x)\,dx$. Find the absolute value of the difference between the new limits of integration.

3. Find the numerical coefficient of x^7 in $\int (1+x)^9\,dx$.

4. Find the smallest positive value of a for which $\int_0^{2\pi} \sin(x)\sin(ax)\,dx = 0$.

5. Let $\int_a^b g(x)\,dx = 24$. If the average value of g on the interval $[a, b]$ is 3, find the value of $b - a$.

6. If $\int (\tan^{13} y)(\sec^6 y)\,dy$ is computed and written out as a sum of powers of $\tan y$, what is the highest power of $\tan y$ that occurs?

7. What is the x-coordinate for which $f(x) = x^2$ attains its average value over the interval $[2, 5]$? Give an exact answer.

8. Find $\int_0^{\pi} (x+1)(\cos(3x))\,dx$. Give answer as a simplified fraction.

9. The area bounded by the x-axis, the line $x = 1$ and the curve $y = \tan x$ is revolved about the y-axis. Find the resulting volume to the nearest hundredth.

10. Given the following values for $f(x)$, using Simpson's Rule estimate $\int_1^3 f(x)\,dx$.

x	1	1.5	2	2.5	3
$f(x)$	2	4	1	−1	2

11. A plate has the shape of the semicircle that is the upper half of the unit circle. The density of the plate at any given point is $x+1$. Find the mass of the plate. Give your answer in terms of π.

12. Find the smallest $a > \frac{\pi}{4}$ for which $\int_{\frac{\pi}{4}}^{a} e^x \sin x\,dx = 0$.

13. Find the area of the triangle-shaped region in the first quadrant whose sides lie along the x-axis, the hyperbola $x^2 - y^2 = 1$, and its asymptote $y = x$. Write "xxxxx" as your answer if the area does not exist. Hint: in polar coordinates, the hyperbola has the equation $r = \frac{1}{\sqrt{\cos 2\theta}}$.

14. Find $\int_0^1 \frac{\sqrt{t}}{1+t}\,dt$. Give your answer in terms of π.

15. Find the length of one "turn" of the Archimedean spiral $r = \theta$, between $\theta = 0$ and $\theta = 2\pi$ to the nearest hundredth.

16. A bucket weighing 15 pounds sits at the bottom of a 40 foot deep well. The rope used to pull the bucket to the top of the well weighs 0.6 lbs./ft. How much work (in foot-pounds) is done to raise the bucket to the top of the well?

17. Approximate the perimeter of the ellipse $\frac{x^2}{4} + y^2 = 1$ to the nearest hundredth.

18. Let $y = \int_0^1 \frac{dx}{x^x}$. Find $\int_0^{\frac{1}{2}} \frac{dx}{(2x)^{(2x)}}$ in terms of y. For the curious, the value of y is the sum of the series $\frac{1}{1^1} + \frac{1}{2^2} + \frac{1}{3^3} + \cdots$.

19. Let $f(x)$ be a function. Differentiate it with respect to x twice. Then antidifferentiate with respect to x three times. Differentiate four more times. Finally, antidifferentiate three more times. The result differs from $f(x)$ by a polynomial. Find the maximum possible degree of this polynomial.

20. Given that $\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$, find $\int_{-\infty}^{\infty} e^{-(x^2+4x+1)}\,dx$. Write "xxxxx" as your answer if the integral does not exist. Give exact answer, if it exists.