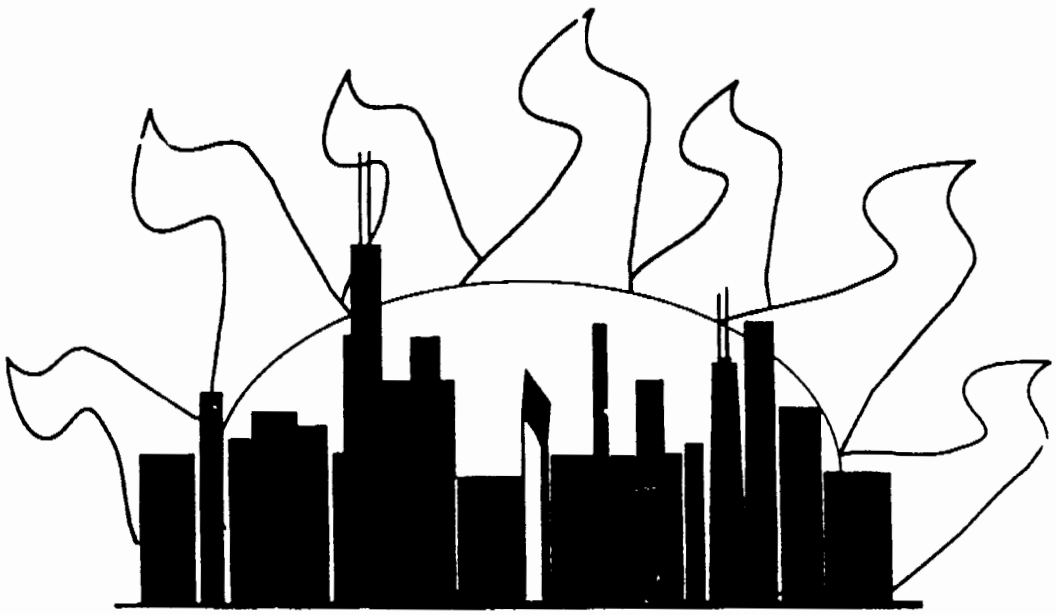
Mu Division **Topic Test 1**

Linear Algebra



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General Instructions:

1. Unless otherwise stated all answers should be written as decimals.
2. If you are asked to give your answer as a fraction, please give your answer in $\frac{a}{b}$ form where a and b are relatively prime.
3. Boldface notation indicates vectors.

Questions

1. Determine the smaller value of k for which $\begin{vmatrix} k & k \\ 4 & 2k \end{vmatrix} = 0$.

2. Determine the absolute value of the sum of the numerical coefficients in the equation of the line determined by the points $P(-1, 3)$ and $Q(4, 6)$. Write the equation in the form $ax + by + c = 0$ with a, b, and c relatively prime integers, then give the absolute value as your answer.

3. If $A = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 1 & -1 \\ 2 & 3 & -2 \\ 0 & 3 & 4 \end{bmatrix}$ find the determinant of $[A][B] - [B][A]$.

4. Find $\|w\|$ if $w = (-3, 1, -2, 4, -5)$. The notation $\|w\|$ denotes the norm or length of w. Give the exact answer.

5. Find the value of $x + y + z + w$ if $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$.

6. If $A = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$ what is the element in the center position of $A^{(-1)}$? Give your answer as a simplified fraction in lowest terms.

7. Suppose $u = (3, 2, 1)$, $v = (5, -3, 4)$ and $w = (1, 6, -7)$. Find $(u + v) \bullet w$.

8. Evaluate $\dfrac{\begin{vmatrix} 1 & 3 & -1 \\ 8 & 5 & 2 \\ -1 & -2 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & 1 & -1 \\ 3 & 8 & 2 \\ 1 & -1 & -3 \end{vmatrix}}$.

9. Determine k so that $\boldsymbol{u} = (1, k, -3)$ and $\boldsymbol{v} = (2, -5, 4)$ are orthogonal.

10. Solve the system $\begin{cases} x + 2z = 7 \\ 2x + y = 16 \\ -2y + 9z = -3 \end{cases}$. Give your answer as $x \wedge y \wedge z$.

11. Sociologists often study the dominance of one group over another. Suppose in a certain society there are four classes: abigwheel, awheel, upancomer and pon. Abigwheel dominates awheel, upancomer and pon. Awheel dominates upancomer and pon. Upancomer dominates pon. Pon dominates none of the other classes. Write a communication matrix, [D], representing these dominance relationships. Use "1" for dominance and "0" for non-dominance. Use rows to represent the dominating class and the columns to represent the dominated classes. We define the power of a class as the sum of the entries in the appropriate row of the matrix $[P] = [D] + [D]^2$. Determine the power of awheel.

12. Find the sum of the entries in the resulting matrix $f(A)$ where $f(x) = x^3 - 3x^2 - 2x + 4$ and $A = \begin{bmatrix} 2 & 2 \\ 3 & -1 \end{bmatrix}$. [Note: The constant "4" in $f(x)$ means "4I" where I is $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.]

13. A sales organization gathers the following data relating the number of salespeople to annual sales.

# of Salespeople	5	6	7	8	9	10
Annual Sales (in millions of $)	2.3	3.2	4.1	5.0	6.1	7.2

Use a best fit line to estimate the annual sales with 14 salespeople. Give your answer rounded to four significant digits.

14. What is the rank of the matrix $\begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & -4 & -5 \\ 7 & 8 & -5 & -1 \\ 10 & 14 & -2 & 8 \end{bmatrix}$?

15. Maximize $z = 2x + 3y$ subject to the following conditions: $\begin{cases} 3x + y \le 6 \\ x + y \le 4 \\ x + 2y \le 6 \\ x \ge 0, \quad y \ge 0 \end{cases}$. Give your answer as a simplified fraction in lowest terms.

16. The Seedy Vin Company produces Riesling, Charbono and Rose wines. There are three procedures for producing each wine and the procedures for production affect the cost of the final product. One procedure allows an outside company to bottle the wine; a second allows the wine to be produced and bottled at the winery; a third allows the wine to be estate bottled. The number of units of wine produced by the Seedy Vin Company by each method is shown by the matrix below:

	Riesling	Charboro	Rose	
W =	2	1	3	outside bottling
	4	3	6	produced & bottled at winery
	1	2	4	estate bottling

Suppose the multiplying factor for the cost in dollars for each method of production is given by the matrix $C = \begin{bmatrix} \text{outside} & \text{winery} & \text{estate} \end{bmatrix} = \begin{bmatrix} 1 & 4 & 6 \end{bmatrix}$. Suppose the production cost, in dollars, of a unit of each type of wine is given by the matrix $D = \begin{bmatrix} 40 \\ 60 \\ 30 \end{bmatrix}$ Riesling / Charbono / Rose.

Find the total cost of production, in dollars, of all three wines.

17. Find to the nearest degree the angle θ, $0° \le \theta \le 180°$, between $u = (1, -2, 3)$ and $v = (3, -5, -7)$.

18. To control a certain type of crop disease, it is necessary to use 23 gallons of chemical A and 34 gallons of chemical B. The dealer can order commercial Spray I, each container of which holds 5 gallons of chemical A and 2 gallons of chemical B and commercial Spray II, each container of which holds 2 gallons of chemical A and 7 gallons of chemical B. How many containers of Spray II should be ordered to obtain exactly the right proportion of chemicals needed?

19. What is the sum of the eigenvalues of $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$?

20. Consider a matrix game with payoff matrix $A = \begin{bmatrix} 2 & -2 & 3 \\ 4 & 0 & -3 \end{bmatrix}$. If $P = \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \end{bmatrix}$ and $Q = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$ are strategies for players R and C, respectively, then determine the expected payoff to R. Give answer as a simplified fraction in lowest terms.