Mu Division   **Topic Test 3**

Number
Theory
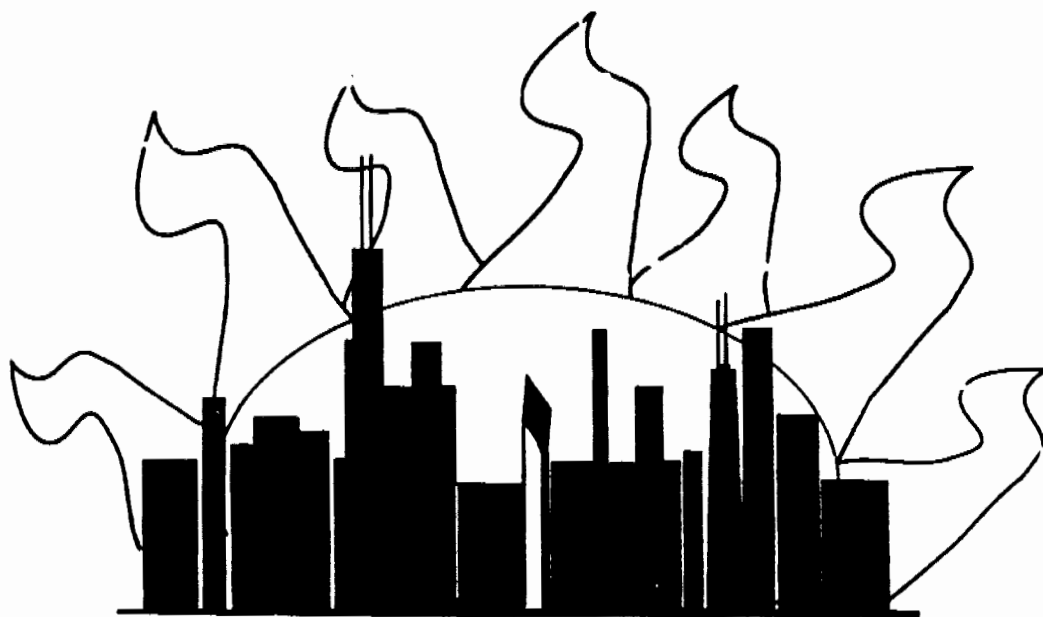
General Instructions:

Unless otherwise stated all answers should be written as decimals.

If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.

Questions

1. Some prime numbers are 1 more than the square of an integer. For example, $5 = 2^2 + 1$. What is the smallest prime greater than 125 that satisfies this condition?

2. Find the sum of the units and tens digits of 3^{1998}.

3. Assume the letters x and y represent digits. The number $66x1_{seven} = 32y5_{nine}$. Find the sum of $2x + 3y$. Give your answer written in base ten.

4. Pairs of consecutive odd integers that are primes are called twin primes. For example, 3 and 5, 11 and 13, and 41 and 43 are twin primes. How many pairs of twin primes are there between 100 and 200?

5. Let $x = .123456789101112...998999$, where the digits are obtained by writing the integers 1 through 999 in order. What is the 1998th digit to the right of the decimal point?

6. If $1333_{four} = 241_x$ find the value of the base x.

7. Find the following sum. Leave your answer in base eight.
    $66_{eight} + 132_{four} + 1011_{two} = \underline{\qquad}_{eight}$

8. Let y be a digit in the base six number system. Find y if $32311_{four} = 4y21_{six}$.

9. There are positive integers that have the properties:
    A. the sum of the squares of their digits is 50 AND
    B. each digit is larger than the one to its left
    What is the product of the digits of the largest integer that has both of these properties?

10. What is the smallest number that is divisible by each of the numbers from one through ten inclusive?

11. What is the value of the units digit of $3^{1998} (7^{1998} - 2^{1998})$?

12. How many two-digit numbers have the property that when they are divided buy the sum of their digits, the quotient is 7? Note: a two-digit number cannot have a 0 as its tens digit.

13. Change 0.846_{twelve} into its equivalent reduced rational fraction written in base ten.

14. The Pythagoreans named a number a perfect number if it was equal to the sum of its proper divisors (divisors that are less than the number itself). For example, the proper divisors of 6 are $\{1,2,3\}$ and $1 + 2 + 3 = 6$, so 6 is a perfect number. What is the next largest perfect number?

15. The sequence of numbers a, b, and c is said to be in a geometric sequence if $b^2 = ac$ in base ten. For what positive base are the numbers $a = 4$, $b = 11$, and $c = 27$ in a geometric sequence?

16. The following equation is written in base five. Solve the equation for x and give your answer in base five.
$$21x - 22 = 12x + 31$$

17. Insert the missing term: 10, 11, 12, 13, 14, 15, 20, 22, 30, ____ , 1100

18. The following fraction has both the numerator and the denominator expressed in base eight. Reduce it in base eight to its lowest terms. Write your answer as a fraction in base eight.

$$\frac{1356}{2070}$$

19. Find the following quotient and write your answer in base two.
      1110101_{two} divided by 1101_{two}.

20. An Armstrong number is an n-digit number equal to the sum of the nth powers of its own digits. Only four integers between 100 and 1000 equal the sum of the cubes of their digits. Three of these are 153, 370, and 407.(Ex: $153 = 1^3 + 5^3 + 3^3 = 1 + 125 + 27 = 153$). Find the fourth such number.