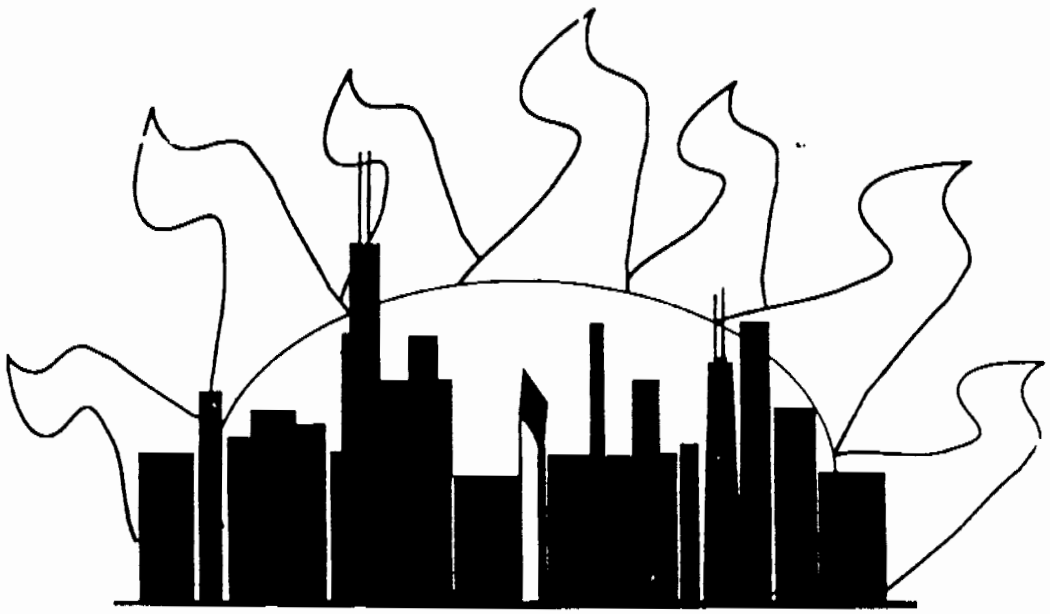
All School Test



Mu Alpha Theta National Convention
Chicago 1998

General Instructions:

Unless otherwise stated all answers should be written as decimals.
If you are asked to give your answer as a fraction, please give your answer in a/b form where a and b are relatively prime.
Answers should be written legibly on the answer sheet.
Your student delegate should bring the answer sheet to his/her topic rest round 3 room and place it in the box next to the head proctor's podium between 8:15 and 9:15 am on Tuesday, August 4.

Questions

1. You need to create a free standing fenced in corral for your cow Clarabell given 100 feet of fencing. Determine the maximum area possible to four significant digits and how many poles you would need to have for this area if you need one pole every 5 feet. Write your answer as an ordered pair (maximum area in sq. ft., number of poles).

2. A "Happy Number" is a number whose sum of the squares of its digits = 1. This process of finding the sum of the squares of its digits is repeated until the sum is a one digit number before the determination of a "Happy Number" is made.
Ex. 19 is happy because
$1^2 + 9^2 = 1 + 81 \rightarrow 8^2 + 2^2 = 64 + 4 = 68 \rightarrow 6^2 + 8^2 = 36 + 64 = 100 \rightarrow 1^2 + 0^2 + 0^2 = 1$.
Find the next "Happy Number" larger than 19.

3. Five men were stranded on a desert island. They gathered all of the coconuts on the island and decided to share them equally the next morning. During the night each man awoke, and fearing that he wouldn't get his fair share, took 1/5 of the remaining coconuts. After each one took his portion of coconuts there was one coconut left over which he gave to the monkey. In the morning the men divided the remaining pile of coconuts evenly and there were no coconuts left over. How many coconuts had the men gathered?

4. Lottie holds 5 tickets for a raffle in which 6 prizes will be awarded. If 100 tickets are sold, what is the probability that Lottie will win exactly 4 prizes? Give your answer as a reduced fraction in lowest terms.

5. Sue and Joe are moonlighting by remodeling bathrooms. Sue is the expert and Joe is assisting her with the heavy work, carrying tiles, arranging them and mixing grout. Sue can, if necessary, work without Joe, but Joe could not do these jobs without Sue. Therefore, Sue makes $150 per day, which is $30 more than joe gets. The total labor cost for doing the bathroom was $2190. Each person worked full days. How many days did each of them work? Give all possible solutions with each answer as an ordered pair (number of days for sue, number of days for Joe).

6. Find the sum of the following numbers:
   the last two (ones and tens) digits of the year that you were born
   the last two digits in which an important event has taken place since 1900
   the number of years that have passed since that important event
   your age as of December 31, 1997

7. Five girls entered a contest:
   Chris was 20 points behind Kay
   Kay was 50 points ahead of Angela
   Angela was 10 points behind Rebecca
   Melody was 30 points ahead of Chris
   Rebecca was 50 points behind melody
   Rank the girls in order, first through last.

8. Kari placed the numbers 0 through 11, inclusive, on cards and arranged them face down in 4 rows with 3 columns. She then asked Elissa to find the hidden numbers using the following clues:
   The top row contains only prime or unique numbers. Its sum is 10. Two of the numbers are odd and the second is larger than the first.
   Row 2 has a sum that is a prime number smaller than 20 and the first number is smaller than the last.
   All the numbers in row 3 are prime. One number is even. Their sum is 18.
   The numbers in row 4 are consecutive when rearranged.
   Column 1 contains numbers that are multiples of 2.
   All numbers in column 2 are odd, and 3 of them are less than or equal to 7.
   In column 3, half of the numbers are odd and half of the numbers are even.
   How were the numbers arranged? Write your arrangement in rows and columns.

9. During an ancient war three prisoners were brought into a room. In the room was a large box containing three white hats and two black hats. Each man was blindfolded, and one of the hats was placed on his head. The men were lined up, one behind the other, facing the wall. The blindfold of the man farthest from the wall was removed, and he was permitted to look at the hats of the two men in front of him. If he knew (not guessed) the color of the hat on his head, he would be freed. However, he was unable to tell. The blindfold was then taken from the head of the next man, who could see only the hat of the one man in front of him. This man had the same chance for freedom, but he, too, was unable to tell the color of his hat. The remaining man then told the guards the color of the hat he was wearing and was released. What color hat was he wearing?

10. A teacher assigned five problems A, B, C, D, and E. Not all students turned in answers to all of the problems. Below is a listing of the percent of students bringing in each problem.

A 46%	A,B 25%	A,B,C 13%	A,B,C,D 7%	A,B,C,D,E 4%
B 40%	B,C 26%	A,B,E 19%	A,B,C,E 8%	
C 43%	C,D 26%	A,D,E 16%	A,B,D,E 9%	
D 38%	D,E 22%	B,C,D 12%	A,C,D,E 11%	
E 41%	A,E 30%	C,D,E 14%	B,C,D,E 6%	

What percent of the students did not hand in any problems? Assume that no students turned in combinations not listed.

11. The County Board of Commissioners which has 20 members, recently had to elect a President. There were three candidates (A, B, and C). On each ballot the three candidates were to be listed in order of preference, with no abstentions. It was found that 11 members, a majority, preferred A over B (thus the other 9 preferred B over A). It was found that 12 members preferred C over A. Given these results, it was suggested that B should withdraw, to enable a runoff election between A and C. However, B protested, and it was then found, after a recount of the votes that 14 members preferred B over C! The Board has not yet recovered from the resulting confusion. Given that every possible order of A, B, and C appeared on at least one ballot, how many board members voted for B as their first choice/

12. The lengths of the legs of a right triangle are 3 inches and 4 inches. Two congruent circles, externally tangent to each other, are drawn inside the triangle, with each circle tangent to the hypotenuse and one of the legs. Find the distance, in inches, between the centers of the two circles. Give your answer as a reduced fraction.

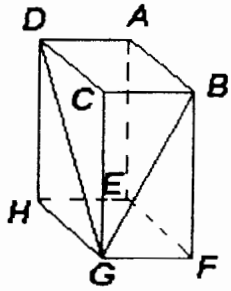
13. Find the perimeters of all non-congruent triangles in which the lengths of all three sides are integral. The cosine of one angle is -0.25 and the length of one of the two sides not opposite this angle is 16.

14. A man is the father of nine children who were born at regular intervals. The sum of the squares of their ages equals the square of the man's age. There were no multiple births and their ages are whole numbers. How old are the children? Write your answer with the ages listed in increasing order separated by commas.

15. Find the coefficient of the term containing $x^{\frac{33}{2}}$ in the expansion of

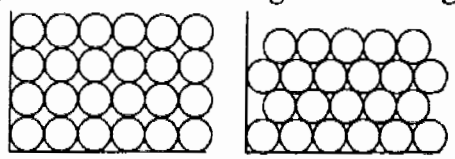
$(x + \sqrt{2x})^{18}$. Give an exact answer.

16. In the rectangular solid below $\triangle DGH=45°$ and $\triangle BGF=60°$. Find the cosine of $\triangle BGD$.



Give an exact answer.

17. A candy manufacturer sells bubble gum cigars in rectangular boxes. Each box holds 120 cigars, set in 8 rows of 15 cigars each. The candy store owner wants to offer the customers a special deal without changing the price per box. The owner decides to pack 15 cigars in the bottom row, 14 in the row above it, 15 in the next row above, etc. similar to the drawing below, hoping to make room for more cigars in the box. Determine how many cigars the owner was able to put into the box using this new configuration and what the difference was in the amount of empty space in the second box compared to the first box. Write your answer as an ordered pair (number of cigars, difference in empty space). Consider the radius of each cigar to be r and the length of each cigar to be l.



18. The sum of the cubes of three consecutive integers equals the cube of the next consecutive integer. What is the sum of the squares of the original three consecutive integers?

19. Ali's Sword shop by the seashore sells sword sheaths, large and small, straight and curved. The only price difference Ali makes is that large sheaths cost twice as much as small ones. Baba the Forgetful needs sheaths for his 40 swords. He buys 40 sheaths in assorted shapes and sizes, and loads them onto his camel. He bought at least one of each kind of sheath and he bought the same number of large straight sheaths as he bought small sheaths. Suddenly he remembers that his 40 swords are all large. Since Ali offers no exchanges or refunds, Baba must buy an additional 70 dinars worth of large sheaths. He bought four times as many straight sheaths as curved sheaths. Only then does Baba remember that all 40 of his swords are also curved, so he returns to Ali's to buy another 50 dinars worth of large curved sheaths. Finally, having bought a total of 100 sheaths, Baba has enough to sheath all 40 of his large curved swords. How many more large sheaths than small sheaths did Ali buy?

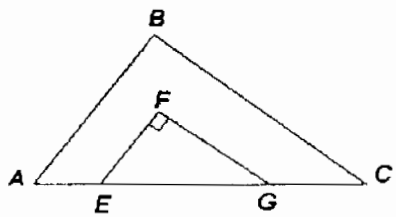
20. Find all of the values of x for which $(x^2 - x + 5)^{x^2 - 9x + 20} = 1$.

21. The County Fish and Game Department issues four types of licenses, for deer, grouse, fish, and wild turkey. Anyone can purchase any combination of licenses. In a recent year (exactly) half the people who bought a grouse license also bought a turkey license. Half the people who bought a turkey license also bought a deer license. Half the people who bought a fish license also bought a deer license. One third of all the people who bought a deer license also bought both grouse and fish licenses. Of the people who bought deer licenses, the same number bought a grouse license as bought a fish license; a similar statement was true of buyers of turkey licenses. Anyone who bought both a grouse and a fish license also bought either a deer or a turkey license, and of these people the same number bought a deer license as bought a turkey license. Anyone who bought both a deer and a turkey license either bought both a grouse and a fish license or neither. The number of people buying a turkey license was equal to the number of people who bought some license but not a fish license. The number of people buying a grouse license was equal to the number of people buying some license but not a turkey license. The number of deer licenses sold was one more than the number of grouse licenses sold. Twelve people bought either a grouse or a deer license (or both). Determine the number of people who bought licenses and the total number of licenses that were sold. Write you answer as the ordered pair, (number of people, number of licenses).

22. The two ends of a string of length 26 units are secured at the points (3, 7) and (3, −3). Assume that the string's length is unaffected by its attachment at the end points. An ellipse can be traced by using a pen to draw the string tight and moving the pen in all directions. Give the equation of this ellipse in the form $\frac{(x-h)^2}{A} + \frac{(y-k)^2}{B} = 1$.

23. Find the sum of the integers 1 through 1000. Multiply this sum by the product of the integers 1 through 1000. From the resulting product subtract the sum of the integers 1 through 1000. Divide this resulting difference by the product of the integers 1 through 1000. Give your answer as the integer closest to the final quotient.

24. In the figure to the right, find the area, in square units, of the shaded region, given that $\triangle ABC \sim \triangle EFG$, $\angle F$ is a right angle, $AB = 6$, $AC = 10$, $FG = 3$. Give your answer as a reduced fraction.



25. Nine men play the positions on a baseball team. Their names are Brown, White, Adams, Miller, Green, Hunter, Knight, Smith, and Jones. Determine from the following information the position played by each man.

a) Brown and Smith each won $10 playing poker with the pitcher.
b) Hunter is taller than Knight and shorter than White, but all three weigh more than the first baseman.
c) The third baseman lives across the corridor from Jones in the same apartment house.
d) Miller and the outfielders play bridge in their spare time.
e) White, Miller, Brown, the right fielder, and the center fielder are bachelors, and the rest are married.
f) Of Adams and Knight, one plays an outfield position.
g) The right fielder is shorter than the center fielder.
h) The third baseman is a brother of the pitcher's wife.
i) Green is taller than the infielders, the pitcher, and the catcher except for Jones, Smith, and Adams.
j) The second baseman beat Jones, Brown, Hunter, and the catcher at cards.
k) The third baseman, the shortstop, and Hunter each made $150 speculating in General Motors stock.
l) The second baseman is engaged to Miller's sister.
m) Adams lives in the same apartment house as his own sister but dislikes the catcher.
n) Adams, Brown, and the shortstop each lost $200 speculating in grain.
o) The catcher has three daughters, the third baseman has two sons, and Green is being sued for divorce.
p) To date, no player is divorced. Both the catcher and the third baseman are married.

26. One bucket contains a gallon of water, another a gallon of alcohol. A cup of alcohol from the second bucket is poured into the first bucket. A cup of the resulting mixture is then poured into the bucket of alcohol. Is there

a) more water in the alcohol bucket than alcohol in the water bucket?
b) more alcohol in the water bucket than water in the alcohol bucket?
c) the same amount of water in the alcohol bucket as alcohol in the water bucket?

27. Find the equation of the line joining the centers of the following conics:

$$x^2 + y^2 - 10x + 4y + 4 = 0$$
$$4x^2 + 9y^2 + 24x - 198y + 1089 = 0$$

Write your equation in the form $Ax + By = C$ where $A > 0$ and A, B are relatively prime integers.

28. A rope over the top of a fence has the same length on each side. The rope weighs 1/3 pound per foot. On one end hangs a monkey holding a banana and on the other hangs a weight equal to the weight of the monkey. The banana weighs 2 ounces per inch. The rope is as long, in feet, as the age of the monkey and the weight of the monkey (in ounces) is as much as the age of the monkey's mother. The combined ages of the monkey and his mother are 30 years. Half the weight of the monkey plus the weight of the banana is 1/4 as much as the weight of the weight and the weight of the rope. The monkey's mother is half as old as the monkey will be when it is 3 times as old as its mother was when she was ½ as old as the monkey will be when it is as old as its mother will be when she is 4 times as old as the monkey was when it was twice as old as its mother was when she was 1/3 as old as the monkey was when it was as old as its mother was when she was 3 times as old as the monkey was when it was 1/4 as old as it is now. How long, in inches, is the banana?

29. Find the composite number(s) in the following set:

31
331
3331
33331
333331
3333331
33333331
333333331

30. What is the smallest positive integral value of n such that $n^2 + n + 41$ is not a prime number?